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A variant of the Hungarian inventory control model

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Abstract

The 'Hungarian inventory control model' was initiated by Prékopa [1965. Reliability equation for an inventory problem and its asymptotic solutions. In: Prékopa, A. (Ed.), Application of the Mathematics to Economics. Publication House of the Hungarian Academy of Science, pp. 317–327] and Ziermann [1964. Application of Smirnov's theorems for an inventory control problem. Publications of the Mathematical Institute of the Hungarian Academy of Science Series B 8, 509–518 (in Hungarian)], where the ordered amount is delivered in an interval, rather than at a time epoch according to some stochastic process and consumption takes place in the same interval. The problem is to determine the minimum level of initial safety stock that ensures continuous consumption, without disruption, in the whole time interval with a prescribed high probability. Prékopa [2006. On the Hungarian inventory control model. European Journal of Operational Research 171, 894–914] has formulated a two-stage model with such interval type processes and probabilistic constraints. In this paper we modify the assumptions of those models and formulate simpler, numerically more tractable models. We also present numerical examples.

Keywords: Inventory control; Stochastic programming; Brownian motion

1. Introduction

The term 'Hungarian inventory control model' refers to a model system, where both the deliveries of the ordered amounts and consumption take place in an interval according to some random processes, rather than at one time epoch. The problem is to determine the minimum level of initial safety stock that ensures continuous consumption, without disruption, in the whole time interval with a prescribed high probability.

The Hungarian inventory control model was initiated by Prékopa (1965) and Ziermann (1964). The original model is of a static and single item type, where the delivered and consumed amounts are assumed to be the same and the mathematical tool used to solve the problem comes from order statistics. In Prékopa (1965) already more general models have been presented and some theorems proved in Prékopa (1973a) have been used to numerically solve the problems. From the later literature in connection with the Hungarian inventory control models we mention the papers by Prékopa and Kelle (1978), Kelle (1984) and the summarizing paper of Prékopa (1980).

Recently Prékopa (2006) has shown that interval type delivery and consumption processes can be combined with classical inventory models. The 'order up to S' model is taken as an example. He also presented dynamic type (two-stage) inventory

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models using interval type delivery and consumption stochastic processes which appear in the Hungarian models. However, the solutions of the obtained nonlinear programming problems are computationally intensive. They involve the solutions of nonlinear decomposition type problems, along with the calculation of the multivariate Dirichlet distribution function and gradient values. A normal approximation to the Dirichlet distribution alleviates the numerical difficulties but still further research is needed to come up with efficient numerical solutions for the problems. Those models are hybrid type stochastic programming models, i.e. both probabilistic constraints and penalties for unsatisfied demands are used.

In the present paper we keep some of the main characteristics of the new models in Prékopa (2006) but introduce simpler, numerically more tractable formulations. We also discuss the numerical solution methods to our problems and present numerical examples.

In Section 2 we recall some earlier results and develop mathematical tools for our model constructions. In Section 3 we formulate a probabilistically constrained multi-item inventory control model with interval type delivery and consumption processes. The consumption process is assumed to be linear with random, normally distributed slope while the expectation of total consumption minus the delivery process is approximated by a Brownian bridge. In Section 4 a two-stage model combined with probabilistic constraints is formulated. We assume that the consumption and delivery processes in connection with the different items are stochastically independent. Research is underway to take stochastic dependence into consideration. Finally, in Section 5 the computational aspects are discussed and numerical examples are presented.

2. Assumptions of the proposed model and its approximation

For the details of the original Hungarian inventory control model and some new variants of it see Prékopa (2006). In connection with the delivery and consumption processes we make the following assumptions:

(a) We assume that delivery takes place during an interval rather than at a single time epoch. We also assume that the delivery process begins τ time after the order is placed and has a duration of time T. Thus, if an order is placed at time 0 then the delivery takes place in the interval $[\tau, \tau+T]$.
(b) Deliveries take place at discrete times the number of which is fixed and designated by n; it can be obtained from past history.
The n delivery times are random and their joint probability distribution is the same as that of n random points chosen independently from the interval $[\tau, \tau+T]$ according to a uniform distribution.
(c) The delivery and consumption processes are stochastically independent.
(d) The consumption of the material is linear with random intensity c. Thus, the total consumption in the time interval $[\tau, \tau+T]$ is cT. We assume that c is normally distributed with mean value μ_c and variance σ_c^2.
(e) The amount delivered in $[\tau, \tau+T]$ is equal to the expected total consumption in that interval. Let $c_0 \stackrel{\text{def}}{=} E(cT)$.
(f) The delivery process can be described by the following model: whenever delivery takes place there is a minimal amount delivered equal to δ. The remaining parts of the n delivery amounts can be described as the lengths of the subsequent intervals obtained by choosing a random sample of size $n-1$ from a population uniformly distributed in the interval $[0, c_0 - n\delta]$.

Assumptions (a)–(c) and (f) have already been introduced in Prékopa (1965) and Kelle (1984) has investigated the case where (d) holds true.

Let $\lambda = \delta n / c_0$ and let $X_n(t, \lambda)$ denote the amount delivered in the time interval (τ, t) where $\tau \leqslant t \leqslant \tau + T$. An amount M of safety stock ensures consumption without disruption, in the same time interval, if and only if $M + X_n(t,\lambda) - c(t-\tau) \geqslant 0$ for any $\tau \leqslant t \leqslant \tau + T$. If we want this to happen with probability at least $1-\varepsilon$ then M has to satisfy the following probabilistic constraint:

$$P\left(\sup_{\tau \leqslant t \leqslant \tau+T} \{c(t-\tau) - X_n(t,\lambda)\} \leqslant M\right) \geqslant 1-\varepsilon, \qquad (2.1)$$

where $0 < \varepsilon < 1$.

Without loss of generality we may assume $T = 1$, therefore $c_0 = E(c) = \mu_c$. If T is not equal to 1 we have to multiply the safety stock level obtained from the model by T to find the actual safety stock level.

For the case of a constant c we have the following: