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A numerical approach to a multi-objective optimal inventory control problem for deteriorating multi-items under fuzzy inflation and discounting

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Abstract

The optimal production and advertising policies for an inventory control system of deteriorating multi-items under a single management are formulated with resource constraints under inflation and discounting in fuzzy environment. Here, the deterioration of the items and depreciation of sales are at a constant rate. Deteriorated items are salvaged and the effect of inflation and time value of money are taken into consideration. The inflation and discount rates are assumed to be imprecise and represented by fuzzy numbers. These imprecise quantities are first transformed to corresponding intervals and then following interval mathematics, the related objective function is changed to respective multi-objective functions. Using Utility Function Method (UFM), the multi-objective problem is changed to a single objective problem. Here, the production and advertisement rates are unknown and considered as control(decision) variables. The production, advertisement and demand rates are functions of time t. The total profit which consists of the sales proceeds, production cost, inventory holding cost and advertisement cost is formulated as an optimal control problem and evaluated numerically using UFM and generalized reduced gradient (GRG) technique. Finally numerical experiment, sensitivity analysis and graphical representation are provided to illustrate the system. For the present model, expressions and graphical results are presented when the rates of advertisement are constant.

Keywords: Multi-item inventory; Optimal production; Deteriorating item; Advertising policies; Fuzzy inflation and discounting

1. Introduction

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. Most of the classical inventory models did not take into account the effects of inflation and time value of money. This has happened mostly because of the belief that inflation and time value of money will not influence the cost and price components (i.e., the inventory policy) to any significant degree. But, during the last few decades, due to high inflation and consequent sharp decline in the purchasing power of money in the developing

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countries like Brazil, Argentina, India, Bangladesh, etc., the financial situation has been changed and so it is not possible to ignore the effect of inflation and time value of money any further. Following Buzacott [1], Misra [2] has extended the approach to different inventory models with finite replenishment, shortages, etc. by considering the time value of money, different inflation rates for the costs. Also Lo et al. [3] developed an integrated production-inventory model with a varying rate of deterioration under imperfect production process, partial backordering and inflation.

In the recent decades, multi-item classical inventory problems were approached by formulating proper mathematical models that considered the factors in real world situations, such as the deterioration of inventory items, depreciation of sales, advertising policies, effects of inflation and time value of money, etc. Deterioration is applicable to many inventory items in practice, like vegetables, rice, medicine, fruits, etc. Recently, Chang and Dye [4], Papachristos and Skouri [5], Maity and Maiti [6] and others formulated EOQ models of deteriorating items with time-varying demand. Also Goyal and Giri [7] have presented a review article on the recent trends in modeling with deteriorating items listing all important publications in this area up to 2001.

Again, some researchers (Cho [8] and others) have assumed depreciation rate of sales as a function of time, t. This assumption is supported by a general fact that, as time goes on, a firm usually faces more competition (thus it may lose its sales at an increasing rate). Again, to boost up the sale, the management goes for advertisement and thus advertisement policy plays an important role in increasing the demand. Also, a promotional cost (cf. Datta et al. [9]) is introduced to provide the advertisement that increase the demand of an item.

In the case of multi-item inventory models, it is possible to study each item separately as long as there is no interactions between the items. However, in general, interaction exist between the items, such as, limited warehouse space, available capital for investment, etc.

The production period of the seasonable products such as winter garments, etc. is normally finite. Moreover, in a production firm, production is discontinued once the level of the stock in godown is such that it is sufficient to fulfill the demand up to the end of the time period.

In a manufacturing system, the physical output (i.e., product) of a firm depends upon the combination of several product factors. These factors are (a) raw material (b) technical knowledge (c) production procedure (d) firm size (e) nature of the organization (f) quality of the product etc. Due to the changes of these factors, production rate and unit production cost are changed too. In the classical production lot size models, both production rate and unit production cost are assumed to be constant and dependent on each other. Several OR scientists developed inventory models for a single product or multiple products taking constant or variable production rate (as a function of demand and/or on hand inventory). In this connection, one may refer to the works of Misra [2], Mandal and Maiti [10]. In their models, the production cost is taken as constant. However, manufacturing flexibility has become much more important to firms and less expensive to acquire. Different types of flexibility in the manufacturing system have been identified in the literature among which volume flexibility is the most important one. Volume flexibility of a manufacturing system is defined as its ability to be operated profitably at different overall output levels. Khouja [11] developed an economic production lot size model under volume flexibility where unit production cost depends upon the raw material used, labour force engaged and tool wear-out cost incurred. Here, unit production cost is a function of production rate. If the production is more, the production related to some constant expenditures are spread over the number of produced units and hence the unit production cost decreases with the increase of produced units. Moreover, some expenditures do not increase linearly with the produced quantity. Bhandari and Sharma [12] extended the work of Khouja [11] including the marketing cost and taking a generalized unit cost function.

However, because of the dynamic nature of the manufacturing environment, the static models may not be adequate in analyzing the behavior of such systems. Dynamic models of production-inventory systems are available in many references (cf. Hu and Loulou [13], Worell and Hall [14], Chandra and Bahner [15], Misra [2], Maity and Maiti [16] and others).

In 1965, the first publication in fuzzy set theory by Zadeh [17] showed the intention to accommodate uncertainty in the non-stochastic sense. After that Bellman and Zadeh [18] defined a fuzzy decision making problem as the confluence of fuzzy objectives and constraints operated by max–min operators. Zimmermann [19] developed a tolerance approach to transform a fuzzy decision making problem to a regular crisp optimization problem and showed that it can be solved to obtain a unique exact optimal solution with highest membership degree using classical optimization algorithms. Recently, fuzzy set theoretic has been applied to several fields like project network, reliability, production planning, inventory problems, etc. Roy and Maiti [20], Mahapatra and Maiti [21] and others have solved the classical EOQ models in fuzzy environment. Now-a-days, some inventory problems have been developed by