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Design of a novel adaptive inventory control system based on the online identification of lead time

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ABSTRACT

In this paper, an adaptation method for the online identification of lead time is incorporated in production–inventory control systems. Based on the lead time estimate, the tuning parameters are updated in real time to improve the efficiency of the system. Combination of the adaptive scheme with a proportional control law is able to eliminate the inventory drift that appears when the actual lead time is not known in advance or when it varies with time. A detailed analysis is provided for the proposed production–inventory system, including a stability analysis and the quantification of its bullwhip effect. Several examples and comparison with state-of-the-art alternative approaches illustrate the efficiency of the system.

1. Introduction

Production planning and inventory control problems have been successfully tackled by control engineering practices over many decades. Classical control theory tools such as Laplace and *z*-transforms, transfer functions, block diagrams and frequency analysis have been utilized by many researchers to describe supply chains, determine efficient replenishment rules and investigate the variance amplification of demand. In a well-celebrated paper published in the early 1980s, Towill (1982) presented the inventory and order-based production control system (IOBPCS) in a block diagram form. This was the first attempt to recast the production–inventory problem into a rigorous control engineering format. The system was subject to many modifications and improvements in subsequent years. In particular, John et al. (1994) introduced the automatic pipeline, inventory and order-based production control system (APIOBPCS), where the order rate (OR) is obtained as a summation consisting of three terms: the average demand (D_{p}), a fraction of the difference (ENS) between the target net stock (TNS) and the actual net stock (NS), and a fraction of the difference (EWIP) between the target work in progress (TWIP) and the actual work in progress (WIP). A number of subsequent publications appeared later that exploited the capabilities of the APIOBPCS framework, improved its performance through parameter optimization (Disney et al., 2000) or proposed modifications. For example the Deizel and Eilon APIOBPCS (DE-APIOBPCS) (Disney and Towill, 2003a, 2006) is a special case of the original method that considers equal fractional amplification in both inventory (ENS, EWIP) signals. An important advantage of DE-APIOBPCS is that stability is guaranteed. An alternative replenishment rule is provided by the automatic pipeline variable inventory and order-based production control system (APVIOBPCS) (Dejonckheere et al., 2003; Lalwani et al., 2006), where the inventory target is not fixed, but changes with demand. The APIOBPCS strategy within vendor-managed inventory (VMI) supply chains has also been studied (Disney and Towill, 2002a, b, 2003b; Dejonckheere et al., 2004). Means of control engineering were also adopted by White (1999), who used a single feedback inventory control loop with a proportional-integral-derivative (PID) controller and a demand-forecasting strategy to determine the order rate

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(OR). Furthermore, predictive control methods have been presented for managing production–inventory systems (Lin et al., 2005; Yu et al., 2004; Wang et al., 2007). These methods are based on the formulation of an online optimization problem that simultaneously minimizes the inventory deficit and the order variation, in order to avoid demand amplification. The behavior of multi-echelon supply chains has also been investigated (Dejonckheere et al., 2004; Hoberg et al., 2006). The effect of information update frequency on the stability of production–inventory systems has been studied by Venkateswaran and Son (2007). A thorough review of the literature concerning the application of control methodologies in production–inventory systems has been provided by Ortega and Lin (2004).

Significant attention in the literature has been given to the so-called bullwhip effect, which is recognized as a major problem in production–inventory systems. The bullwhip effect is directly related to the variance amplification between demand and order rate. Some earlier studies (Meters, 1997; Chen et al., 2000) have identified numerous factors that contribute to this undesired phenomenon. Among them, non-zero lead times and poor forecasting of the demand signal are two major causes of bullwhip. Statistical analysis of the bullwhip effect for different forecasting methods can be found in Zhang (2004) and in Sun and Ren (2005). Statistical analysis was also used by Kim et al. (2006) where systems involving stochastic lead times were studied. Significant progress has been achieved towards the estimation and suppression of the bullwhip effect (Dejonckheere et al., 2003, 2004; Disney and Towill, 2003a; Disney et al., 2004; Towill et al., 2007) based on the APIOBPCS replenishment rule and various control engineering approaches, including proportional-integral (PI) controllers, cascade control (Lin et al., 2004) and PID controllers (Rivera and Pew, 2005).

Besides stability issues and the bullwhip effect, the efficiency of the different control strategies as far as the response of the net stock signal is concerned is an additional important subject for investigation. A major problem in many classical control schemes, is that inventory deficit (difference between inventory target and actual inventory level) cannot be eliminated. This phenomenon, usually denoted as inventory drift, appears when the true lead time is not known with accuracy. In order to overcome this problem, PI controllers with adaptive capabilities have been proposed in the past (Towill et al., 1997; Evans et al., 1998). More recently, Disney and Towill (2005) proposed a simple variation of the original APVIOBPCS that allows the elimination of inventory drifts, under proportional (P) control. The difference in the new scheme, called estimated pipeline variable inventory and order-based production control system (EPVIOBPCS), is that the transfer function WIP(z)/OR(z) is based on the estimated lead time and not on the actual one. However, the characteristic equation of the closed loop transfer function between demand $D(s)$ and actual net stock NS(s) NS(z)/$D(z)$ involves both the actual lead time and the lead time estimate and thus stability conditions are not fixed, but they depend on the values of those two parameters. This increases substantially the tuning effort, given that the actual lead time is not known with accuracy. The system designer should consider any lead time that could actually be the case in the future and thus several stability conditions corresponding to all possible actual lead times must be satisfied simultaneously.

In this work, an adaptive control methodology is applied to the APIOBPCS in order to ensure that the actual lead time is estimated with accuracy and lead time changes can be tracked online. The recursive prediction error method (RPEM) (Söderström and Stoica, 1989; Ljung, 1999) is used to identify the lead time, based on historical data regarding order rate and received finished products. The lead time estimate is then used to define the target WIP (TWIP), eliminating in this way the inventory drift, under proportional control. The proposed adaptive configuration is compared with the standard APIOBPC system and with a modified version of the EPVIOBPC system and is found superior, especially when lead time varies with time. Furthermore, a simple condition is derived to guarantee the stability of the proposed system.

The rest of the paper is formulated as follows: in Section 2 the lead time adaptation mechanism is described in details. In Section 3, the proposed adaptive control strategy is introduced. Section 3 contains a detailed analysis of the proposed system based on transfer functions, the terminal value theorem and the derivation of a stability condition. Section 4 presents simulated results on different test cases, where the performance of the proposed system is compared with state-of-the-art alternative approaches. The paper ends with the concluding remarks in Section 5.

2. Lead time identification method

In many production–inventory systems, the production process is modeled by a pure delay unit, with a discrete transfer function equal to z^{-T_p}, where T_p is the lead time. In the proposed methodology we follow the same assumption using a different formulation. More specifically, we pose a bound $T_{p,\max}$ on the initially unknown lead time, meaning that the actual lead time T_p is an integer that satisfies $0 \leqslant T_p \leqslant T_{p,\max}$. We also assume that there is an additional unit delay between the time the order is received and the time where execution of the order starts. Thus, we can model the production dynamics by the following difference equation:

$$\begin{aligned} R(t) = {} & \theta_0\,\mathrm{OR}(t-1) + \theta_1\,\mathrm{OR}(t-2) \\ & + \cdots + \theta_{T_{p,\max}-1}\,\mathrm{OR} \times (t - T_{p,\max}) \\ & + \theta_{T_{p,\max}}\,\mathrm{OR}(t - T_{p,\max} - 1) \end{aligned} \tag{1}$$

where the model coefficients are

$$\theta_i = \begin{cases} 1, & \text{if} \quad i = T_p \\ 0, & \text{if} \quad i \neq T_p \end{cases} \tag{2}$$

In Eq. (1) OR(t) is the order rate, while $R(t)$ is the production rate discrete time signal. Eq. (1) can easily lead to the transfer function between production rate and