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Optimal Model and Algorithm for Multi-Commodity Logistics Network Design Considering Stochastic Demand and Inventory Control

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Abstract: A simultaneous approach that incorporates inventory control decision into facility location model is proposed, which is used to solve the multi-commodity logistics network design problem. Based on the assumption that the stochastic demands of the retailers are normal distributed, a non-linear mixed integer programming model that simultaneously described the inventory decision and the facility location decision is presented, in which the objective is to minimize the total cost that including location costs, inventory costs, and transportation costs under the certain service level. The combined simulated annealing (CSA) algorithm is developed to solve the problem. The model and effectiveness of the algorithm are clarified by the computational experiments.

Key words: multi-commodity; logistics network design; stochastic demand; optimization model; simulated annealing algorithm

1 Introduction

In a high competitive environment, the manufacturing companies must pay close attention to their inventory management. To optimize their inventory system, the companies should solve two critical problems. First, they must select the proper places that the commodities saving, namely, the sites and the number of stocking locations or logistics nodes (LNs). Second, they must determine the amount of commodities to maintain in each LN. So in the logistics network design problem, the facility location problem and inventory decision problem are two key subproblems and both of them are highly related. But in many literatures, the above two problems always were studied as the facility location problem [1−3] and the inventory control problem[4−5] separately. The decision-making results in incompatibility and inconformity at different levels, which could affect the rationality of the final strategy decisions.

In addition, the demands of the retailers for the commodities are always uncertain in the real world, but in the research on the logistics network design problem, the demands were always considered as a deterministic variables in order to simplify the analysis and modeling. Furthermore, the companies should maintain a certain stock to satisfy the stochastic demands as far as possible. They are required to control their inventory cost because the inventory cost is increasing following the inventory amount, so the companies must select the scientific inventory policies. Based on the assumption that the stochastic demand of the each retailer is normal distributed, the problem that integrated the facility location problem and inventory control problem is studied in

this article, which could increase the rationality and scientificity of the decisions.

For the single commodity logistics network design problem considering the inventory cost, the literatures [6-8] ignored many factors which have influence on the inventory cost, and only added the cost as the non-linear function of the commodity quantity to the objective function; the literature [9] studied the joint location-inventory problem under two special cases: the variance of demand was proportional to the mean and the demand had zero variance, and restructured the model into a set-covering integer programming model; the literature [10] developed a more efficiency algorithm for the special cases in the literature [9]; the literature [11] analyzed the transportation cost considering the vehicle routing in the logistics network, but the order number was considered as a continuous variable in the formulation derivation; the literature [12] investigated the trade-offs problem between the service level and service cost making use of the existed model in the literatures [9-10], and proposed a weighting method and a heuristic solution approach based on genetic algorithms to solve the problem.

The literatures that studied the logistics network design problem with multi-commodity are few for the present at home and abroad. The literature [13] simplified the inventory cost of the commodities as in the literatures [6-8] and proposed the Lagrange algorithm to solve the problem; the literatures [14–15] regarded the inventory cost as the linear function of commodity quantity; the literature [16] developed the model framework of multi-commodity dynamic capacitated facility location and reported on their computa-

tional experience with standard mathematical programming software, but the inventory cost in the model was a linear function of demand quantity too.

2 Assumptions and notations

To formulate the model, the following notations are used: i denotes the candidate LN site, $i = 1, 2, \cdots, N$; j denotes the retailer, $j = 1, 2, \cdots, M$; l denotes the commodity, $l = 1, 2, \cdots, L$. F_i is the fixed cost of locating at candidate LN site i; V_i denotes the storage capacity of candidate LN i; RP_i^l is the reorder point of the inventory policy in LN i for commodity l, and when the inventory amount of commodity l at LN i decreases to RP_i^l, then an order is triggered; Q_i^l is the order quantity per order; λ_l is the occupation of storage capacity per unit commodity l; d_j^l and u_j^l are the mean and the standard deviation of the demand at retailer j for commodity l; D_j^l and U_j^l are the mean and the standard deviation of the demand at candidate LN i for commodity l; LT_i^l is the lead time (in days) from supplier to LN i for commodity l, namely, the supplier takes the required lead time to fulfill an incoming order from LN i for commodity l; H_i^l is the inventory holding cost per unit commodity l at LN i; O_i^l is the fixed cost of places an order for commodity l at LN i; R_i^l is the cost to ship per unit commodity l from the supplier to candidate LN i; T_i^l is the elapsed time between two consecutive orders for commodity l at LN i; C_{ij}^l is the distribution cost of per unit commodity l between the LN i and the retailer j; θ_1 and θ_2 are the weighted factors associated with transportation cost and inventory cost, respectively; α is the united service level in the system, $0 < \alpha < 1$, that is to say, the fill rate of all demands for all commodities must not less than α in all LNs; K is the planning horizon; γ is bank interest rate and β is discount rate (calculated by γ); P is the maximum number for the LNs that allowed to locate.

And set the binary variables as:

$$X_i = \left\{ \begin{array}{l} 1, \text{ if a LN setup on site } i \\ 0, \text{ otherwise} \end{array} \right.$$

$$Y_{ij}^l = \left\{ \begin{array}{l} 1, \text{ if LN } i \text{ services retailer } j \\ \quad \text{ for commodity } l \\ 0, \text{ otherwise} \end{array} \right.$$

Note that when $X_i = 0$, there is no commodities should pass LN i and vice versa.

Some rational assumptions are proposed as following:

(1) Each factory in the network produces only one commodity;

(2) The demand of each retailer for each commodity is uncertain and satisfies a normal distribution. Moreover, all the demands are independent.

(3) Each demand is serviced by only one LN, namely, the demand could not be partitioned.

(4) LN i performs an inventory control policy (RP_i^l, Q_i^l), that is to say, a fixed quantity RP_i^l is ordered to the supplier, once the inventory quantity falls to or below a reorder point Q_i^l, and the factory could satisfy the demand of retailer i for commodity l after lead time LT_i^l.

(5) All LNs in the network should have the same service level, namely, the fill rates for the demands in all LNs are identical.

Based on the above assumptions, we get:

$$D_i^l = \sum_{j=1}^{M} d_j^l Y_{ij}^l, \quad i = 1, 2, \cdots, N. \quad (1)$$

$$U_i^l = \sum_{j=1}^{M} u_j^l Y_{ij}^l, \quad i = 1, 2, \cdots, N. \quad (2)$$

The fixed location costs of LNs are disposable and the other costs are invested per day, so in order to keep the consistency of all cost in unit time, the location cost should be shared in day in the planning horizon. The absorption rate could be computed as following:

$$\beta = \frac{1}{365} \sum_{h=1}^{K} \frac{\gamma}{(1+\gamma)^h - 1}. \quad (3)$$

3 System cost analysis

According to the above assumptions, the LN i performs an inventory policy (RP_i^l, Q_i^l) to meet the stochastic demand pattern. But even the order is triggered, the commodities should receive after LT_i^l days. So once an order is submitted, the inventory commodities should cover the demand produced in lead time LT_i^l with a certain probability α. The probability α is known as the given service level of the system. So the level-of-service constrains at LN i for commodity l can be expressed as follows:

$$P(D(LT_i^l) \le RP_i^l) = \alpha; \quad \forall i, l. \quad (4)$$

where $D(LT_i^l)$ is the random demand quantity during the lead time at LN i for commodity l.

Based on the inventory policy (RP_i^l, Q_i^l) and the assumption of normally distributed demand, RP_i^l can be determined as follows:

$$RP_i^l = D_i^l LT_i^l + Z_\alpha \sqrt{LT_i^l} \sqrt{U_i^l}. \quad (5)$$

where Z_α is the value of the standard normal distribution, which denotes the uniform service level and is identical in the network. For simplicity, we let $Z = Z_\alpha$ in following analysis. So the average holding cost rate in each LN i for commodity l, based on the expression (5), could be written as:

$$H_i^l Z \sqrt{LT_i^l} \sqrt{U_i^l} + H_i^l Q_i^l / 2. \quad (6)$$

The first term $H_i^l Z \sqrt{LT_i^l} \sqrt{U_i^l}$ in (6), is the average expenditure associated with safety stock kept at LN i. The second term $H_i^l Q_i^l / 2$ is the average expenditure incurred due to the holding the order quantity Q_i^l, which is the inventory used to cover the demand arisen between two successive orders. Thus, the operation cost during this period at LN i for commodity l is given by:

$$R_i^l Q_i^l + O_i^l + (H_i^l Z \sqrt{LT_i^l} \sqrt{U_i^l} + H_i^l Q_i^l / 2) T_i^l. \quad (7)$$

Then we divide the expression (7) by T_i^l ($T_i^l = Q_i^l / D_i^l$), the operation cost rate incurred at LN i for commodity l is given by the following expression:

$$\left(R_i^l + \frac{O_i^l}{Q_i^l} \right) D_i^l + H_i^l Z \sqrt{LT_i^l} \sqrt{U_i^l} + H_i^l Q_i^l / 2. \quad (8)$$