Electric Power Systems Research 57 (2001) 189–194

ELECTRIC POWER SYSTEMS RESEARCH

www.elsevier.com/locate/epsr

Steady-state phase-coordinate model for induction machines

D. Hoadley, S. Moorthy, M. Al-Dabbagh

Department of Electrical Engineering, Royal Melbourne Institute of Technology, PO Box 2476V, Melbourne, Vic. 3001, Australia

Received 6 March 2000; accepted 5 July 2000

Abstract

A phase-coordinate model for induction machines is developed, to facilitate load-flow studies of small hydro- or wind generators connected to distribution systems. The phase-coordinate method allows ready analysis of system unbalance, and the model is applicable to both induction motors and generators under these conditions. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Phase-coordinate analysis; Load flow; Induction machines

1. Introduction

In the analysis of unbalanced three-phase networks, phase coordinate methods [1] may offer advantages over the traditional symmetrical component methods. The symmetrical component approach is elegant and simple when applied to a balanced system with unbalanced loads, but when there is unbalance in the network itself, the method loses its decoupling effect and becomes cumbersome. In this case, the direct phase coordinate method becomes simpler.

When applying the phase coordinate methods to the study of rural untransposed distribution lines with connected induction generators, as may be found with wind turbines or small hydro-electric schemes, or with large induction motor loads, a three-phase model of the induction machine is required.

In a loadflow study, modelling the induction machine as a PV type node is not desirable since the machine does not maintain voltage magnitude as would a synchronous machine. Neither can it be readily modelled as a PQ type node, because its P–Q ratio is variable. Since induction machine reactive power varies with terminal voltage (and shaft power) it is desirable to model this behaviour directly.

2. Model derivation

The phase coordinate representation of a general three-phase admittance is shown in Fig. 1. The voltages and currents are related by the equation

$$\mathbf{I}_{abc} = \mathbf{Y}_{abc}\mathbf{V}_{abc} \tag{1}$$

where $\mathbf{V}_{abc} = [V_a \quad V_b \quad V_c]^T = [(V_1 - V_4)(V_2 - V_5)(V_3 - V_6)]^T$ and $\mathbf{I}_{abc}$ represents the currents flowing in each phase.

For balanced components, $\mathbf{Y}_{abc}$ may conveniently be formed by transformation of symmetrical component values, where they are available. The sequence components are related by the equation

$$\mathbf{I}_{012} = \mathbf{Y}_{012}\mathbf{V}_{012} \tag{2}$$

where

$$\mathbf{Y}_{012} = \begin{bmatrix} y_0 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & y_2 \end{bmatrix}$$

in which it is assumed that there is no mutual coupling between the sequence admittances. The phase voltage and currents are related to the sequence components by the relations

$$\mathbf{V}_{abc} = \mathbf{T}\mathbf{V}_{012} \tag{3}$$

$$\mathbf{I}_{abc} = \mathbf{T}\mathbf{I}_{012} \tag{4}$$

PII: S0378-7796(01)00079-7

where

$$\boldsymbol{T}=\begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix},\quad a=e^{j2\pi/3}$$

So from Eqs. (1)–(4) we obtain

$$\mathbf{I}_{\mathrm{abc}}=\mathbf{TY}_{012}\mathbf{V}_{012}=\mathbf{TY}_{012}\mathbf{T}^{-1}\mathbf{V}_{\mathrm{abc}} \tag{5}$$

$$\mathbf{Y}_{\mathrm{abc}}=\mathbf{TY}_{012}\mathbf{T}^{-1}=\frac{1}{3}\mathbf{TY}_{012}\mathbf{T}^{*}$$

$$=\frac{1}{3}\begin{bmatrix}y_0+y_1+y_2 & y_0+ay_1+a^2y_2 & y_0+a^2y_1+ay_2\\ y_0+a^2y_1+ay_2 & y_0+y_1+y_2 & y_0+ay_1+a^2y_2\\ y_0+ay_1+a^2y_2 & y_0+a^2y_1+ay_2 & y_0+y_1+y_2\end{bmatrix} \tag{6}$$

In the context of this paper, it is convenient to think of $\mathbf{Y}_{\mathrm{abc}}$ as the sum of two 3×3 matrices: $\mathbf{Y}_0$, which contains all of the zero sequence terms, and $\mathbf{Y}_{\pm}$, which contains the remainder.

$$\mathbf{Y}_0=\frac{1}{3}\begin{bmatrix}y_0 & y_0 & y_0\\ y_0 & y_0 & y_0\\ y_0 & y_0 & y_0\end{bmatrix},$$

$$\mathbf{Y}_{\pm}=\frac{1}{3}\begin{bmatrix}y_1+y_2 & ay_1+a^2y_2 & a^2y_1+ay_2\\ a^2y_1+ay_2 & y_1+y_2 & ay_1+a^2y_2\\ ay_1+a^2y_2 & a^2y_1+ay_2 & y_1+y_2\end{bmatrix}$$

2.1. Formation of induction machine matrix

The standard single phase induction machine equivalent circuit is shown in Fig. 2. This is taken as the positive sequence representation of the machine. For an induction generator, rotor speed is above synchronous speed, so slip s is negative. This causes effective rotor resistance also to be negative, so it appears to act as a power source rather than a power sink. The negative sequence circuit, shown in Fig. 3, is almost the same.

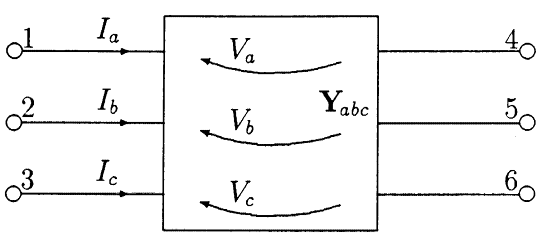

Fig. 1. General three-phase admittance.

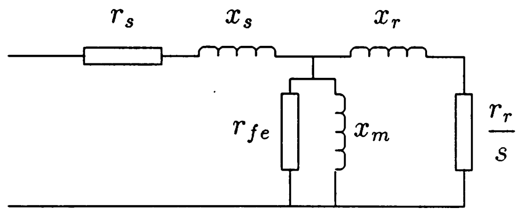

Fig. 2. Induction machine equivalent circuit — positive sequence.

The stator and magnetising impedances remain the same, but since the negative sequence field rotates backwards, the effective slip is $2-s$. Zero sequence values y_{s_0}, y_{m_0}, and y_{r_0} generally are not measured, since induction machines typically do not have earthed star-points, so there is no path for zero-sequence currents to flow.

The equivalent three-phase representation of the machine, connected either in delta or in star, but without the star point brought out or earthed, is shown in Fig. 4. From this, we obtain

$$\begin{bmatrix}\mathbf{I}_{123}\\ \hline \mathbf{I}_{456}\\ \hline \mathbf{I}_{789}\end{bmatrix}=\left[\begin{array}{c|c|c}\mathbf{Y}_{s_{abc}} & -\mathbf{Y}_{s_{abc}} & \mathbf{0}\\ \hline -\mathbf{Y}_{s_{abc}} & \mathbf{Y}_{s_{abc}}+\mathbf{Y}_{m_{abc}}+\mathbf{Y}_{r_{abc}} & -\mathbf{Y}_{m_{abc}}-\mathbf{Y}_{r_{abc}}\\ \hline \mathbf{0} & -\mathbf{Y}_{m_{abc}}-\mathbf{Y}_{r_{abc}} & \mathbf{Y}_{m_{abc}}+\mathbf{Y}_{r_{abc}}\end{array}\right]\begin{bmatrix}\mathbf{V}_{123}\\ \hline \mathbf{V}_{456}\\ \hline \mathbf{V}_{789}\end{bmatrix} \tag{7}$$

2.2. Matrix reduction

Since nodes 7, 8 and 9 are shorted together, and there is no injected current, the following relations hold.

$$I_7+I_8+I_9=0 \tag{8}$$

$$V_N=V_7=V_8=V_9 \tag{9}$$

and the induction machine matrix can be simplified by adding the last three rows and columns

$$\begin{bmatrix}\mathbf{I}_{123}\\ \hline \mathbf{I}_{456}\\ \hline 0\end{bmatrix}=\left[\begin{array}{c|c|c}\mathbf{Y}_{s_{abc}} & -\mathbf{Y}_{s_{abc}} & \begin{matrix}0\\0\\0\end{matrix}\\ \hline -\mathbf{Y}_{s_{abc}} & \mathbf{Y}_{s_{abc}} & \begin{matrix}(-y_{m_0}-y_{r_0})\\(-y_{m_0}-y_{r_0})\\(-y_{m_0}-y_{r_0})\end{matrix}\\ \hline 0\ 0\ 0 & (-y_{m_0}-y_{r_0})\ (-y_{m_0}-y_{r_0})\ (-y_{m_0}-y_{r_0}) & 3(y_{m_0}+y_{r_0})\end{array}\right]\begin{bmatrix}\mathbf{V}_{123}\\ \hline \mathbf{V}_{456}\\ \hline V_N\end{bmatrix} \tag{10}$$

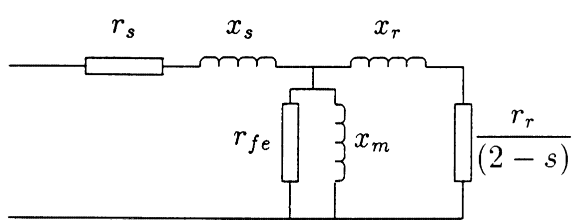

Fig. 3. Induction machine equivalent circuit — negative sequence.