Applied Mathematical Modelling 35 (2011) 497–505

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Inventory control by different service levels

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ARTICLE INFO

Article history:
Received 9 February 2009
Received in revised form 5 July 2010
Accepted 7 July 2010
Available online 13 July 2010

Keywords:
Inventory/production
Operating characteristics
Dynamic pricing
Multiple service levels

ABSTRACT

This paper examines the multiple period inventory control problem of a single product with multiple (two) prices, depending on service level, in which optimal pricing and ordering decisions are made in each period. Traditional inventory and pricing models consider only single products, single prices, and single service levels. However, this research paper finds that a seller can improve inventory control and revenue by offering multiple prices depending on service level. This research considers a single product with multiple (two) pricing policies corresponding to service level as follows: if the customer is willing to delay the shipment, he/she will be offered a lower regular price. Otherwise, the customer will pay the regular price plus extra charges for express service. In this paper, I show the following: (1) there is an optimal pricing and replenishment policy that can control inventory and (2) there exists a finite threshold for inventory levels such that if the inventory level at the beginning of each period is higher than the threshold, the customer will be offered the express service at the regular price, without any extra charge.

1. Introduction

In many cases, you are offered a low price if you are willing to accept slower service. For instance, consider online stores such as Amazon.com and Buy.com. If you are willing to accept delayed shipping of your order, you are commonly offered a lower price than that for faster shipment. In other words, you pay more for the faster shipment of your order. In this paper, to identify how to control the inventory problem herein, single products whose inventory levels and selling prices are reviewed periodically and decided dynamically are considered as having two prices, depending on service level. The two service levels consist of express service and regular service. In many practical cases, a customer will respond to the offered service level depending on the price in the following sense. When ordering an item from a seller, some customers might be willing to delay shipment if they are offered a lower price. This provides the seller with advance demand information to apply to the next production/ordering period so that inventory can be controlled more efficiently. Other customers require the ordered item to be shipped as soon as possible, even if that necessitates paying an extra charge. This is called express service. Therefore, depending on the customer's willingness to delay the shipment, service level can be classified into regular and express level and discriminated by price. Suppose that at period t there are two types of customers. One orders an item with regular service and the other orders an item with express service. The items will be shipped as shown in Fig. 1. The model in this paper is similar to the model outlined in Ref. [1] except that Ref. [1] considers single products with a single pricing policy, whereas the model in this paper considers single products with a multiple (two) pricing policy depending on the service level. Ref. [2] shows the properties of an optimal pricing schedule depending on the customers own surplus. Ref. [2] assumes that total amount of demand for all service levels does not depend on the price. However, this paper assumes that total amount of demand depends on the price. Ref. [2] uses the demand model in which each customers demand level depends on their willingness-to-pay for shipment delay. This paper, however, uses that in which customers proportionally react to

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doi:10.1016/j.apm.2010.07.015

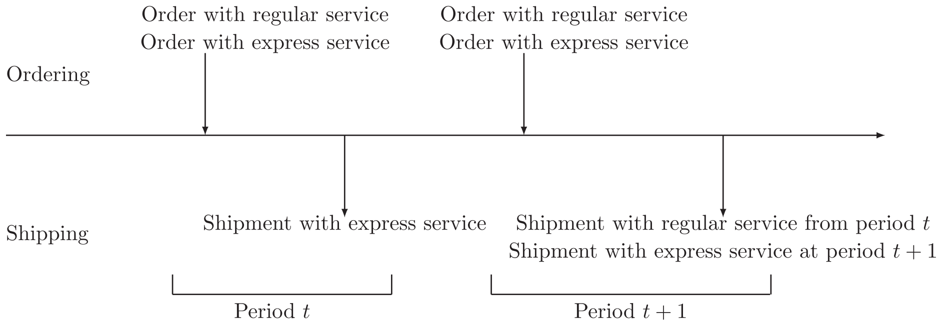

Fig. 1. Timeline of ordering and shipping.

given prices by moving dynamically between service levels: if the price for the express service increases, some portion of express service customers moves to regular service. Otherwise, some portion of regular service customers moves to express service. Refs. [3,1] consider the dynamic inventory control model with single priced products and single service levels, showing the (s,S,p)-policy and the base stock policy as the optimal policies, respectively. Ref. [3] considers that the fixed order cost for each replenishment occurs, whereas Ref. [1] does not. They assume that customers will select only one service level, whereas the customer in this paper may select one of a number of service levels. Ref. [4] assumes that each customers order will be shipped at some future time after the order is placed. This delay is referred to as demand lead-time, and it is equivalent to service level in this paper. However, this demand lead-time is fixed for any given order, resulting in only one service level choice. Each customer in Ref. [4] cannot select this lead-time, whereas each customer in this paper can select the lead-time depending on the price.

2. Assumptions and notations

Notations:

1. $c \equiv$ per unit purchase or production cost.
2. $p_R \equiv$ price charged for regular service for all periods on $[c, \bar{p}_R]$.
3. $p_{E,t} \equiv$ price charged for express service for period t.
4. $p_t = p_{E,t} - p_R \equiv$ extra charge for the express service at period t on $[0, \bar{p}]$.
5. $\epsilon_t \equiv$ random term with a known distribution.
6. $D_{E,t} = d_{E,t}(p_t, \epsilon_t) \equiv$ demand for the express service.
7. $D_{R,t} = d_{R,t}(p_t) \equiv$ demand for the regular service.
8. $x_t \equiv$ inventory level at the beginning of period t before replenishment.
9. $y_t \equiv$ inventory level at the beginning of period t after replenishment.
10. $h_t(I) \equiv$ inventory (or backlogging) cost at the end of period t.
11. $H_t(y_t, p) \equiv E_{\epsilon_t}[h_t(y_t - d_{R,t-1}(p_{t-1}, \epsilon_{t-1}) - d_{E,t}(p, \epsilon_t))]$.
12. For convenience, E_{ϵ_t} is written as just E.

Assumption 1. Replenishment becomes available instantaneously.

Assumption 2. Excess demand is backlogged.

In the area of Economics and Operations Research, the demand has been frequently assumed to be a concave and decreasing function or a linear and decreasing function in the price. Moreover expected demand is assumed to be finite and strictly decreasing in the price. Since this decreasing demand, which is either concave or linear function in price, can be negative as the price is sufficiently large, another assumption has been made such that the set of feasible price level is confined in the finite interval to avoid the negative demand Refs. [1–3,5]. Thus for technical reasons, the following assumption regarding the demand function is made.

Assumption 3. For all $t = 1, 2, \ldots, T$, the function $d_{E,t}(p, \epsilon_t) = a(p_R, \epsilon_t) - bp$ is a linearly decreasing function in $p \in [0, \bar{p}]$, and $d_{R,t}(p, \epsilon_t) = bp$ is a linearly increasing function in p, where $a(p_R, \epsilon_t)$ is the possible maximum demand and a nonincreasing linear function of $p_R \in [c, \bar{p}_R]$, where values for $\bar{p}$ and $\bar{p}_R$ are taken such that $a(\bar{p}_R, \epsilon_t) - b\bar{p}$ is positive w.p.1, and b is deterministic.

Assumption 3 is established from the following perspective. Suppose that the price for the regular service is set at p_R. Total number of customers including the regular service and express service in period t, $D_{R,t}(p, \epsilon_t) + D_{E,t}(p, \epsilon_t)$, is equal to $a(p_R, \epsilon_t)$ and all customers will be divided into either regular service or express service depending on the price difference p. This implies that initially all customers are enticed at regular service and then decide which service to select depending on the price difference $p = p_E - p_R$ between regular and express service. The larger the price difference will be, the less the cus-