Applied Soft Computing 11 (2011) 103–110

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Modeling and simulation of chaotic phenomena in electrical power systems

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ARTICLE INFO

Article history:
Received 23 December 2007
Received in revised form 30 October 2009
Accepted 15 November 2009
Available online 18 November 2009

Keywords:
Nonlinear system
Chaos
Hoff bifurcation
Double scroll equation
Dynamical system
Limit sets
Power system instability

ABSTRACT

Modeling and simulation of nonlinear systems under chaotic behavior is presented. Nonlinear systems and their relation to chaos as a result of nonlinear interaction of different elements in the system are presented. Application of chaotic theory for power systems is discussed through simulation results. Simulation of some mathematical equations, e.g. Vander Pol's equation, Lorenz's equation, Duffing's equation and double scroll equations are presented. Theoretical aspects of dynamical systems, the existence of chaos in power system and their dependency on system parameters and initial conditions using computer simulations are discussed. From the results one can easily understand the strange attractor and transient stages to voltage collapse, angle instability or voltage collapse and angle divergence simultaneously. Important simulation results of chaos for a model three bus system are presented and discussed.

1. Introduction

Chaotic phenomena have been drawing extensive attention in various fields of natural science [1]. Recent developments in nonlinear system theories allow one to understand and analyze several complex behaviors in power systems. Nonlinear phenomena such as bifurcation and chaos in power systems has been observed in the power system networks during the past few years [2]. Disturbances in power system causes change in parameters which result in the system exhibiting chaotic behavior. When chaos breaks, it enters into different instability modes, which causes the power systems to exhibit instability which needs to be avoided. Most of the physical systems in nature are nonlinear and as a result powerful mathematical tools are required for analysis [3,4]. It is desirable to make linear assumptions whenever a compromise can be obtained between the simplicity of analysis and accuracy of results.

Chaotic phenomena are one type of un-deterministic oscillation existing in deterministic systems. They are related to random, continuous and bounded oscillation and not dynamically stable and may face serious problems from an operation view point. The Hoff bifurcation and chaos limit the load-ability of the power system and are unwanted phenomena [5]. For their complexity, mechanism of chaotic phenomena is very little known up to now. There is no generally accepted definition of chaos. Hence is called strange attractor. Discovery of chaos enhances our understanding of complex and unpredictable behaviors arising from a wide variety of systems in engineering and sciences, mainly in nonlinear systems research. Also, study on chaotic phenomena is one important part of power system stability studies [6,7]. In this paper the numerical simulation of the mathematical relations for chaos occurring in power systems have been simulated. The behavior of the system under various operating conditions is presented.

The paper is organized as follows. Theoretical formulation and mathematical representation of chaos is given in Section 2. Section 3 provides the steady-state behavior of nonlinear systems. Modeling of chaotic behavior in power systems is described in Section 4. Section 5 provides the implementation aspects of the chaos. Chaos and instability in power systems is provided in Section 6. Important conclusions are given in Section 7.

2. Nonlinear dynamical systems

Three types of dynamical systems are presented with some useful facts from the theory of differential equations [1,8].

2.1. Autonomous dynamical systems

An nth order autonomous dynamical system is defined by the state equation

$$\dot{x} = f(x) \quad x(t_0) = x_0 \tag{1}$$

where $\dot{x} = dy/dt$ and $x(t) \in \Re$ are the state at time t and f. $\Re \rightarrow \Re$ is called the vector field.

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doi:10.1016/j.asoc.2009.11.001

The solution to (1) with initial condition x_0 at time $t=0$ is called a trajectory and is denoted by $\Phi_t(x_0)$. The dynamical system (1) is linear if $f(x)$ is linear.

2.2. Non-autonomous dynamical systems

An nth order non-autonomous dynamical system is defined by the time varying state equation

$$\dot{x} = f(x, t) \quad x(t_0) = x_0 \tag{2}$$

The vector field f depends on time and, unlike the autonomous case, the initial time cannot arbitrarily be set to zero.

The solution to (2) passing through x_0 at time t_0 is $\Phi_t(x_0, t_0)$. The system is linear if f is linear with respect to x.

If there exist a $T>0$ such that $f(x, t) = f(x, t+T)$, for all x and all t, the system is said to be time periodic with period T. The smallest such T is called the minimal period.

An nth order time periodic non-autonomous system can always be converted to an $n+1$ order autonomous system by appending an extra state $\theta{:=}2\pi t/T$.

2.3. Useful facts about dynamical systems

It is assumed that for any finite t, Φ_t is a '*diffeomorphism*'. This is not a restrictive assumption and has several consequences.

(i) $\Phi_t(x) = \Phi_t(y)$, if and only if $x=y$. Hence, trajectories of autonomous systems are uniquely specified by their initial condition.
(ii) $\Phi_t(x, t_0) = \Phi_t(y, t_0)$, if and only if $x=y$. This implies that given the initial time, a trajectory of a non-autonomous system is uniquely specified by the initial state; however, if $t_0 \neq t_1$, it is possible that $\Phi_t(x, t_0) = \Phi_t(y, t_1)$ and $x \neq y$ showing that, unlike autonomous systems can intersect.
(iii) The derivative of a trajectory with respect to the initial condition exists and is nonsingular. It follows that for t and t_0 fixed, $\Phi_t(x_0, t_0)$ is continuous with respect to the initial condition.

2.4. Discrete-time dynamical system

The general form of state model for a multivariable discrete-time system is

$$x[(k+1)T] = f(x(k)), \quad k = 0, 1, 2, \ldots, \tag{3}$$

where $x(KT)$ is state vector, $u(KT)$ is input vector. For the analysis and design of discrete-time dynamical system Z-transform is needed. $x(KT)$ is called the state and f maps the state $x(KT)$ to the next state and so on.

3. Steady-state behavior and limit sets of nonlinear systems

Steady-state refers to the asymptotic behavior as $t \to \infty$. It is required that the steady-state be bounded. The difference between the solution and its steady-state is called transient. The set of all limit points of x is called the set $L(x)$ of x. Limit sets are closed and invariant under Φ_t. A limit set L is attracting if there exist an open neighborhood U of L such $L(x) = L$ for all $x \in U$. The basin of attraction $B(L)$ of an attracting set L is defined as the union of all such neighborhoods U. Every trajectory starting in $B(L)$ tends towards L as $t \to \infty$.

In a stable linear system, there is only one limit set. Hence, the steady-state behavior is independent of initial condition and it makes sense to speak, for example, of the sinusoidal steady-state. In a typical nonlinear system, however, there can be several limit sets. In particular, there can be several attracting limit sets each with a different basin of attraction. The initial condition determines in which limit set the system eventually settle [3,9].

Four steady-state behaviors associated with the nonlinear systems are discussed in detail.

1. Equilibrium points
2. Periodic solutions
3. Quasi-periodic solutions
4. Chaos

3.1. Equilibrium points

The most general description of nonlinear control system is$\dot{X}(t) = f(X(t), U(t))$where $X \in \Re$ is the state vector, $U \in \Re$ is the control vectors. If state feedback is used, the control variables $u_1, u_2, u_3, \ldots, u_m$ are the functions of state variable X, expressed as $U=U(X)$. By using this vector function, the general mathematical definition of nonlinear control system in the form are obtained as

$$\dot{X}(t) = f(X) \tag{4}$$

taking care, $f(X)=0$

This is the solution of the algebraic equation. A nonlinear system may have multiple equilibrium points because equation set $\dot{X}(t) = f(X)$ usually has multiple solutions.

A non-autonomous system typically does not have equilibrium points because the vector field varies with time.

An equilibrium point x of an autonomous system is a constant solution of (1), $\Phi_t(x) = x$ for all t.

A simple is the damped pendulum equation

$$\dot{x} = y \tag{5}$$

$$\dot{y} = -ky - \sin x \tag{6}$$

which is a second-order autonomous system with an infinite of equilibrium points at

$$(x, y) = (k, \pi, 0), \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

The limit set for an equilibrium point is just the equilibrium point itself.

3.2. Periodic solutions

$\Phi_t(x*, t_0)$ is a periodic solution if, $\Phi_t(x*, t_0) = \Phi_{t+T'}(x*, t_0)$.

For all t and some minimal periods $T'>0$.

Consider Vander Pol's differential equation:

$$\dot{x} = y \tag{7}$$

$$\dot{y} = \mu(1 - x^2)y - x \tag{8}$$

which describe physical situations in many nonlinear systems.

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0 \tag{9}$$

Fig. 1 shows the different types of limit cycle behavior of nonlinear system. It can be observed from Fig. 1(a) that initially $|x| \gg 1$ the damping factor $(x^2 - 1)$ has large positive value. The system behaves like an over damped system with consequent decrease of the amplitude of x; and damping factor decreases as a result system state finally enters a limit cycle. On the other hand, if initially $|x| \ll 1$, the damping is negative hence the amplitude of x increases till the system state again enters the limit cycle. On the other hand, if the path in the neighborhood of a limit cycle diverges away from it, it indicates that the limit cycle is unstable as shown in Fig. 1(b).

Consider, for example, the Vander Pol's equation with the sign of its damping term reversed, i.e.

$$\ddot{x} + \mu(1 - x^2)\dot{x} + x = 0 \tag{10}$$