Available online at www.sciencedirect.com
ScienceDirect
Electrical Power and Energy Systems 30 (2008) 415–427
ELECTRICAL POWER & ENERGY SYSTEMS
www.elsevier.com/locate/ijepes

Hierarchical algorithms of functional modelling for solution of optimal operation problems in electrical power systems

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Accepted 18 February 2008

Abstract

This paper presents foundations of the optimization method intended for solution of power systems operation problems and based on the principles of functional modeling (FM). This paper also presents several types of hierarchical FM algorithms for economic dispatch in these systems derived from this method. According to the FM method a power system is represented by hierarchical model consisting of systems of equations of lower (subsystem) levels and higher level system of connection equations (SCE), in which only boundary variables of subsystems are present. Solution of optimization problem in accordance with the FM method consists of the following operations: (1) solution of optimization problem for each subsystem (values of boundary variables for subsystems should be determined on the higher level of model); (2) calculation of functional characteristic (FC) of each subsystem, pertaining to state of subsystem on current iteration (these two steps are carried out on the lower level of the model); (3) formation and solution of the higher level system of equations (SCE), which gives values of boundary and supplementary boundary variables on current iteration. The key elements in the general structure of the FM method are FCs of subsystems, which represent them on the higher level of the model as "black boxes". Important advantage of hierarchical FM algorithms is that results obtained with them on each iteration are identical to those of corresponding basic one level algorithms.

Keywords: Optimization; Hierarchical algorithms; Functional modeling; Distributed processing; Economic dispatch; Optimal power flow

1. Introduction

One of the promising trends of modern progress in methods and algorithms destined for planning, operating and controlling power generation and transmission systems is development of methods and algorithms oriented to solution of these problems on distributed computer systems. The most important advantages presented by this approach is an opportunity to obtain efficient parallelism of data processing and small amount of data communication necessary for handling these tasks. It makes this approach very attractive compared with traditional centralized organization of solution based on formation and processing of single system of equations pertaining to a system as a whole.

In this paper, we present theoretical foundations of hierarchical method intended for solution of optimization problems in electrical power systems and based on the principles of functional modeling. At present time there exist all technical conditions needful for practical implementation of algorithms oriented to parallel solution of several systems of equations (each of them pertaining to one of subsystems in large system) on various computers and exchange of information among them. The main problem in this field until nowadays was creation of such methods of optimization which on one hand would be able to use this organization of calculations and data communication and on the other hand would yield on each iteration same

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doi:10.1016/j.ijepes.2008.02.001

results which could be obtained from solution of single system of equation for system taken as a whole.

The main difficulty in development of optimization methods which could meet these requirements is calculation of boundary variable values on each iteration. The problem is that values of these variables should be found from two conditions, which must be satisfied simultaneously: optimality condition for each variable and condition of compatibility for values of boundary variables when they are computed in two subsystems adjacent to each border between subsystems [4,5].

In spite of some achievements in this field the problem of formation such systems of equations, from which values of boundary variables satisfying these conditions could be found and which could be efficiently solved in distributed system, was till now far from having been solved.

In this paper, we present theoretical foundations of hierarchical method intended for solution of optimization problems in electrical power systems and based on the principles of functional modelling (FM). The main objective pursued in creation of this method was solution in general form of the problem we mentioned above, i.e. determination of the values of boundary variables, satisfying conditions of optimality and compatibility. This problem can be solved by application of the general approach of the FM method. Principles of the FM method were presented previously in [1–3,9] and are stated as follows:

(1) Representation of subsystems in large system by functional characteristics (FCs). FCs are input–output characteristics in which vectors of boundary variables of one kind are considered as input variables and boundary variables of another kind as output variables. These characteristics are obtained while meeting all constraints within subsystem.
(2) Building of a model as a hierarchical structure, consisting of interconnected systems of equations. In this structure subsystems are represented by lower level systems of equations. A higher level system of equations represents borders between subsystems (boundary nodes).
(3) On the lower level each subsystem is presented as an open system influenced by input variables on it's borders. On this level two problems are solved: calculation of internal variables in subsystems and determination of their FCs. The problem of the higher level is calculation of the values of boundary variables. This problem should be solved through formation and solution of the system of connection equations (SCE), comprising equations obtained from expressions presenting conditions of compatibility (matching conditions) for values of boundary variables computed in adjacent subsystems.

This formulation applies to solution by the FM method of power system modeling and analysis problems. It is necessary to modify and generalize this formulation if we want to develop an optimization method based on the principles of FM.

In [3] it has been shown in general features how the FM method can be applied to solution of power system optimization problems. A FM type algorithm for solution of economic dispatch problem was also presented for simple case when only characteristics of stations are taken into account. Neither active power losses nor network equations were included in the model.

In this paper we present: (1) general formulation of power system optimal operation problem as it looks from the point of view of FM method; (2) derivation of main equations forming the higher level system of equations – SCE and (3) general structure of model and organization of the solution process in the FM algorithm.

Optimality equations for boundary variables forming the SCE are derived in this paper in general form from Lagrange function, representing a system as a set of subsystems and including equations of matching conditions for boundary variables when they are determined in adjacent subsystems.

General structure and convergence properties of the FM algorithms presented here are based on the principle of optimality for hierarchical models formulated in this paper. Due to application of this principle calculation results obtained by any FM algorithm on each iteration and final results are identical to the results of basic sequential algorithm from which it is derived.

It is important to note that the problems considered in the first part of this paper are used mainly as material for derivation of the FM algorithms and for demonstration of the general logic of the FM method. These problems in themselves are of small interest from theoretical and practical points of view. The purpose of this paper is the presentation of the FM method as a method creating new opportunities to overcome serious difficulties and problems which arise in application of the concept of central control to large interconnected power systems. These interconnected systems often include large power systems of independent countries and it causes great organizational and political problems for implementation of centralized optimization and control in these systems.

Important advantage of the general structure of the FM method and the FM algorithms is the presence of the upper level model (SCE) in which only boundary variables of subsystems are represented. In economical sense it means that in this model the problem of markets can be solved when the optimal values of power flows between subsystems are calculated. No information about internal parameters and variables of subsystems is needed for forming and solution of this problem.

From the point of view of computational efficiency the FM algorithms perform as a distributed Gaussian elimination when the FCs of subsystems are calculated and as a distributed back substitution when the downward moves in subsystems are executed. Convergence properties of these algorithms are the same as in basic sequential algorithms.