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Study of the periodic ferroresonance in the electrical power networks by bifurcation diagrams

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ABSTRACT

The principal contribution of this article is to determine the existence zones of the various periodic ferroresonant modes (fundamental, harmonic and subharmonic) intervening in the electrical power network. The bifurcation diagrams are used for this purpose. To be able to plot a bifurcation diagram of a particular solution, it is initially necessary to locate this solution and then to follow it according to a bifurcation parameter. The developed computation code, resulting from the implementation of the Galerkin method jointly with the pseudo-arclength continuation method, has proved to be a powerful and reliable tool to construct these bifurcation diagrams. Indeed, it enables the electrical power network operators to better understand the problems of the phenomenon which had been observed in its network and to foresee new ferroresonance cases.

Applied to the single-phase ferroresonant circuits, both series and parallel, we have been able to easily calculate the existence zones of these modes, as well as the stability limits of the system on ranges of broad variation of the parameters.

Several results obtained numerically by software MATLAB, as from the real cases, are presented and commented upon.

1. Introduction: phenomenon modelling and adapted mathematical framework

The ferroresonance is a nonlinear resonance phenomenon that can affect the electrical transmission and distribution networks [1–3]. It indicates all oscillating phenomena, generally periodic, which particularly appear on all networks at capacitive dominant (single-phase or polyphase) in interaction with a ferromagnetic and saturable element (unloaded transformer).

This phenomenon is characterized by the possibility of existence of several stable steady states for a given configuration and parameter values. These various operating modes, except the normal state, are obviously undesirable. They lead indeed either to out-of-tolerance overvoltages with values several times in excess of the network nominal voltage (case of fundamental or harmonic ferroresonance), or to overcurrents not less dangerous for the material (case of subharmonic ferroresonance) [1,4–6].

Modelling a ferroresonant circuit leads to a system of nonlinear differential equations depending upon various physical parameters, of the form:

$$\frac{dx}{dt} = F(x, t, \lambda) \tag{1}$$

$x \in \mathbb{R}^m$ is the vector of the state variables (flow, voltages and currents) appearing on the network, $\lambda \in \mathbb{R}^p$ is the vector of the physical parameters of network (lines length, impedances, supply voltage, etc.).

The nonlinearity of the function F with respect to its variables x_i, $1 \leqslant i \leqslant m$, is related to the nonlinear characteristic of the transformer. Indeed, the ferroresonance occurs when the transformer functions in the zone of magnetic saturation.

From a mathematical point of view, it is known that a nonlinear system of the form (1) can behave in an unexpected way and thus it appears normal that the solutions (flux, voltages and currents) can present brutal changes. In varying the value of certain parameters of the network, the solution form can change abruptly; several solutions can appear or disappear, lose or gain stability, etc. The terms singularity and bifurcation are then used.

Currently, it is admitted that the mathematical framework adapted to the study of these dynamic systems is the bifurcation theory or the catastrophe theory [7–10]. The main tool within this framework is the continuation methods. They consist in following a particular solution while gradually continuing a physical parameter whose influence we study. The obtained curve is called bifurcation diagram. It will be possible then to determine all the

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bifurcation diagrams of a particular mode, and thus to highlight the various changes of the type of solutions (bifurcations) met. This enables us to obtain an overview of the phenomenon and thus to answer the questions of the electric power network operators anxious to guarantee the reliability of the electricity supply.

In this study, we are interested uniquely in steady periodic solutions. We show that the Galerkin method, used jointly with a continuation method [14–18], enables us to directly calculate the bifurcation diagrams of the periodic states. Our goal is to determine the existence zones of the various periodic ferroresonant modes, intervening in the ferroresonant circuits (series and parallel), according to these physical parameters λ_j, $1 \leqslant j \leqslant p$, called bifurcation parameters.

In the beginning, we present the studied ferroresonant system (series or parallel single-phase circuit) in mathematical form within the meaning of the Galerkin method. We carry out a network modelling based on the equivalent Thevenin circuit in order to reduce the number of network reactive elements and thus the complexity of the differential equations system which describes its behavior. Then, we will see how to use the Galerkin method in the pseudo-arclength continuation method [14–18] for systematic construction of the bifurcation diagrams of the periodic ferroresonance. These two methods are described in detail and are established in the environment of the software MATLAB. Finally, we expose, on real cases of ferroresonant circuit, the wealth of information acquired by the proposed tool. Several results of continuation (relating to the series and parallel ferroresonance) concerning the fundamental mode, harmonics 3, 5 and 7 modes and subharmonics 3, 5, 7, 9 and 5/3 modes are presented. We detail the degree of influence of the supply voltage, the losses, the circuit capacitance, etc., on the ferroresonance phenomenon. This parametric study is of a great interest for the network operators, because the bifurcation diagrams enable us to provide a global vision of the phenomena and their occurrence domains according to the most significant parameters for the ferroresonance.

2. Mathematical equations of steady state and numerical methods

As the solutions in steady state are generally periodic, one seeks a formulation in frequency mode of the problem. For that, we adopt an equations formulation by the Galerkin method [14–16].

2.1. Galerkin method

2.1.1. Principle

This method consists in finding an approximate periodic solution $x(t)$ of the nonlinear differential Eq. (1) by minimizing the error associated with this solution. The idea is to seek this solution in the form of limited Fourier series:

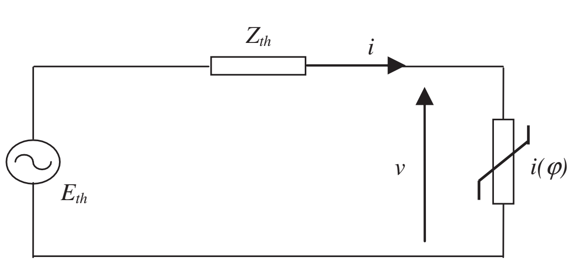

Fig. 1. Equivalent study circuit.

$$x(t) = a_0 + \sum_{k=1}^{n} (a_k \cdot \cos(k\omega t) + b_k \cdot \sin(k\omega t)) \quad (2)$$

with n is selected high so that the precision obtained on the solution will be very close to that obtained by the transient numerical simulation.

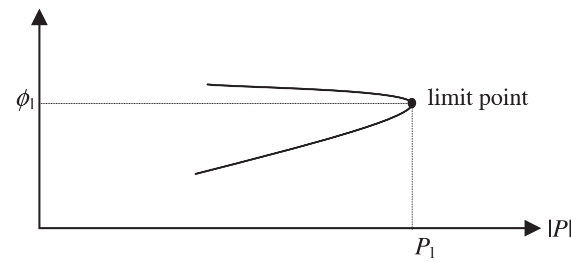

Fig. 2. Example of curve of a bifurcation diagram branch in the limit point proximity.

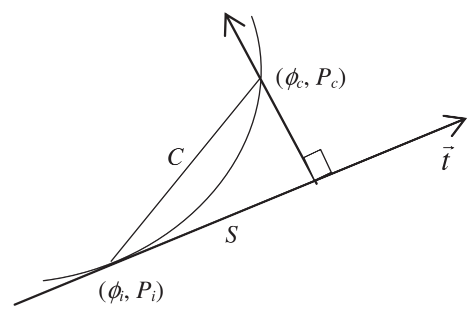

Fig. 3. Principle of the pseudo-arclength continuation method.

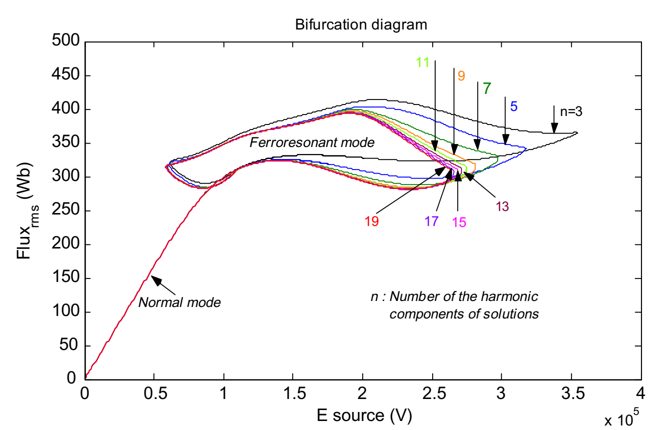

Fig. 4. Influence of harmonic component number n on the periodic solutions.

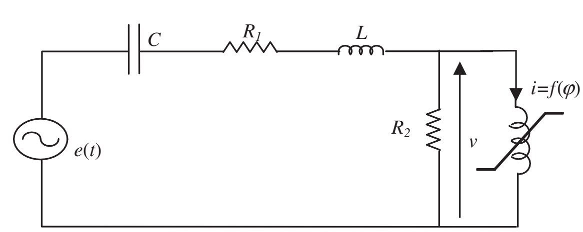

Fig. 5. Series, single-phase, nonlinear ferroresonant circuit.