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A belief-rule-based inventory control method under nonstationary and uncertain demand

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ABSTRACT

This paper is devoted to investigating inventory control problems under nonstationary and uncertain demand. A belief-rule-based inventory control (BRB-IC) method is developed, which can be applied in situations where demand and demand-forecast-error (DFE) do not follow certain stochastic distribution and forecasting demand is given in single-point or interval styles. The method can assist decision-making through a belief-rule structure that can be constructed, initialized and adjusted using both manager's knowledge and operational data. An extended optimal base stock (EOBS) policy is proved for initializing the belief-rule-base (BRB), and a BRB-IC inference approach with interval inputs is proposed. A numerical example and a case study are examined to demonstrate potential applications of the BRB-IC method. These studies show that the belief-rule-based expert system is flexible and valid for inventory control. The case study also shows that the BRB-IC method can compensate DFE by training BRB using historical demand data for generating reliable ordering policy.

1. Introduction

Most inventory control problems occur in industrial, distribution and service situations where demand is nonstationary and uncertain (Treharne & Sox, 2002). The nonstationarities may arise due to the following reasons: (1) a multi-stage of product life-cycle (Metan & Thiele, 2008; Song & Zipkin, 1993), (2) technological innovation and reduced product life (Bertsimas & Thiele, 2006a; Giannoccaro, Pontrandolfo, & Scozzi, 2003; Song & Zipkin, 1993), (3) seasonal effects (Karlin, 1960; Zipkin, 1989), (4) volatile customer tastes (Bertsimas & Thiele, 2006a), (5) changes in economic conditions (Song & Zipkin, 1993), and (6) exchange rate fluctuations (Scheller-Wolf & Tayur, 1997). It is inevitable that future demand will come from a distribution that differs from what governs historical demand (Scarf, 1958).

Previous studies in inventory control with nonstationary demand mainly focused on stochastic methodologies with specialized demand models. For instance, demands in successive periods were characterized by different known distributions (Bollapragada & Morton, 1999; Gavirneni & Tayur, 2001; Karlin, 1960; Morton, 1978; Tarim & Kingsman, 2006; Veinott, 1966), nonstationary Markov decision processes (Iida, 1999; Song & Zipkin, 1993; Treharne & Sox, 2002) and autoregressive, moving average or mixed autoregressive-moving average processes (Johnson & Thompson, 1975; Lee, Padmanabhan, & Whang, 1997; Lee, So, & Tang, 2000; Raghunathan, 2001). Besides, Kurawarwala and Matsuo (1996) proposed an integrated framework for forecasting and inventory management for short-cycle products. Demand-price related problems were also formulated using stochastic methodologies (Federgruen & Heching, 1999; Gallego & van Ryzin, 1994).

However, it is not always realistic to get accurate knowledge about demand such as stochastic distribution and time series characteristics in real-life inventory problem (Bertsimas & Thiele, 2006a; Petrovic, Petrovic, & Vujosevic, 1996). Inventory control strategies generated on the basis of unrealistic assumptions can result in poor performances. There is therefore a need to investigate alternative non-probabilistic inventory control strategies with uncertain and limited information about demand. Fuzzy mathematical programming (Dey & Chakraborty, 2009; Li, Kabadi, & Nair, 2002; Petrovic et al., 1996; Roy & Maiti, 1997; Yao & Su, 2000) and robust counterpart optimization (RCO) (Bertsimas & Thiele, 2006a, 2006b) have been studied to solve uncertain inventory problems by completely discarding the stochastic premise. In these studies, a series of forecasting demands is modeled in forms of fuzzy sets or intervals, and optimal policies are obtained on the basis of the finite future planning periods without taking into account historical demand. Moreover, fuzzy logical systems are proposed to solve inventory control problems with fuzzy forecasting demand (Hung, Fang, Nuttle, & King, 1997; Kamal & Sculfort, 2007; Leung, Lau, &

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Kwong, 2003), but the fuzzy rules reported in the literature are set by only taking into account qualitative expert knowledge without considering quantitative expert knowledge and historical demand. Another traditional approximation approach is to transform uncertain forecasting demand into single-point value based on the manager's knowledge and preferences (Gen, Tsujimura, & Zheng, 1997), then use stochastic approximation to generate "optimal" inventory control policies. This approach does not take full account of demand uncertainty, so the derived "optimal" policies can be hardly reliable.

Besides the nonstationary and uncertain concerns, most of the models are based on a definite planning horizon into the future, which results in that the order quantity for the forthcoming period can be significantly affected by forecasts for distant periods. However, the forecasting for distant periods is hardly reliable (Mellichamp & Love, 1978). Based on these considerations, we propose a belief-rule-based inventory control (BRB-IC) method according to current inventory, historical demand data and necessary short-term forecasting demand. The method is developed from the decision support mechanism of belief-rule-based inference methodology – RIMER (Yang, Liu, Wang, Sii, & Wang, 2006; Yang, Liu, Xu, Wang, & Wang, 2007) which is derived on the basis of evidential reasoning (ER) approach (Yang, 2001; Yang & Sen, 1994; Yang & Singh, 1994; Yang & Xu, 2002a; Yang & Xu, 2002b) and rule-based expert system. RIMER is a modeling and inference scheme under uncertainty with a belief-rule structure. It has been applied to graphite content detection (Yang et al., 2006, 2007), pipeline leak detection (Chen, Yang, Xu, Zhou, & Tang, 2011; Xu et al., 2007; Zhou, Hu, Xu, Yang, & Zhou, 2011; Zhou, Hu, Yang, Xu, & Zhou, 2009), clinical guideline (Kong, Xu, Liu, & Yang, 2009), nuclear safeguards evaluation (Liu, Ruan, Wang, & Martinez, 2009), consumer preference prediction (Wang, Yang, Xu, & Chin, 2009), new product development (Tang, Yang, Chin, Wong, & Liu, 2011), system reliability prediction (Hu, Si, & Yang, 2010), and gyroscopic drift prediction (Si, Hu, Yang, & Zhang, 2011). The BRB-IC method provides a transparent knowledge-based framework for inventory control, and can handle various kinds of uncertain information. It enables experts and decision-makers (DMs) to intervene the construction and updating of the belief-rule-base (BRB) using their judgmental knowledge. The method is easy to understand and implement and requires little computational efforts.

The paper is organized as follows: In Section 2, the basic model formulations for backorder and lost sales cases are briefly described. In Section 3, the BRB-IC method is described in terms of information transformation in ER framework, constructing BRB, initializing BRB, BRB-IC inference approaches and training BRB. In Section 4, a numerical example and a case study are examined to demonstrate potential applications of the BRB-IC method. The paper is concluded in Section 5.

2. Inventory model formulations

We consider a single-echelon periodic review inventory problem, the objective of which is to determine order quantity that maximizes total profit (equivalent to minimize total cost). The cost elements include purchase, holding and shortage (or backorder) costs, and the setup cost is negligible relative to other factors. Review period T and lead time L are assumed to be constant. One cycle of the order and arrival process requires $L + T$ periods of time. Demand forecasting, arrival of goods, review, determining order quantity, and placing an order are assumed to take place at the beginning of the period sequentially. Customer demand is assumed to take place during the period. Cost and profit are calculated at the end of the period. Inventory equations for both backorder and lost sales cases are given as follows (Axsater, 2007; Zipkin, 2000).

(A) Backorder case.

Inventory level IL_n is decided by period $n-1$'s inventory level IL_{n-1} and real demand D_{n-1} as well as period $n-L$'s order quantity Q_{n-L}.

$$IL_n = \begin{cases} IL_{n-1} - D_{n-1} + Q_{n-L}, & n > L \\ IL_{n-1} - D_{n-1}, & n \leqslant L \end{cases} \tag{1}$$

Backlogged quantity BL_n is given as following

$$BL_n = \begin{cases} 0, & n \leqslant L \vee (n > L \wedge IL_{n-1} - D_{n-1} > 0) \\ D_{n-1} - IL_{n-1}, & n > L \wedge IL_{n-1} - D_{n-1} < 0 \wedge IL_{n-1} \\ & -D_{n-1} + Q_{n-L} > 0 \\ Q_{n-L}, & IL_{n-1} - D_{n-1} + Q_{n-L} \leqslant 0 \end{cases} \tag{2}$$

In-transit inventory IT'_n before period n's order quantity Q_n is placed equals to the last $L-1$ periods' total order quantity, or

$$IT'_n = \begin{cases} \sum_{i=n-L+1}^{n-1} Q_i, & n > L \\ \sum_{i=1}^{n-1} Q_i, & n \leqslant L \end{cases} \tag{3}$$

In-transit inventory IT_n after period n's order quantity Q_n is placed equals to the sum of IT'_n and Q_n, or

$$IT_n = IT'_n + Q_n = \begin{cases} \sum_{i=n-L+1}^{n} Q_i, & n > L \\ \sum_{i=1}^{n} Q_i, & n \leqslant L \end{cases} \tag{4}$$

Inventory position IP_n is the sum of IL_n and IT_n, or

$$IP_n = IL_n + IT_n = IL_n + IT'_n + Q_n \tag{5}$$

Total cost TC is the sum of products' purchase cost, holding cost and shortage penalty cost. Total profit TP is the difference between total revenue TR and total cost TC,

$$TC = \sum_{n=1}^{N} (p_2 \cdot Q_n + h \cdot [IL_n - D_n]^+ + p \cdot [IL_n - D_n]^-) \tag{6}$$

$$TR = \sum_{n=1}^{N} p_1 \cdot (\min([IL_n]^+, D_n) + BL_n) \tag{7}$$

$$TP = TR - TC = \sum_{n=1}^{N} (p_1 \cdot (\min([IL_n]^+, D_n) + BL_n) - p_2 \cdot Q_n - h \cdot [IL_n - D_n]^+ - p \cdot [IL_n - D_n]^-) \tag{8}$$

where p_1 is the sales price per item, p_2 the purchasing price per item, h the holding cost per item per period, and p the backorder penalty cost per item per period.

(B) Lost sales case.

On-hand inventory OH_n is given by

$$OH_n = \begin{cases} OH_{n-1} - D_{n-1}, & n \leqslant L \ \wedge \ OH_{n-1} > D_{n-1} \\ 0, & n \leqslant L \ \wedge \ OH_{n-1} \leqslant D_{n-1} \\ OH_{n-1} - D_{n-1} + Q_{n-L}, & n > L \ \wedge \ OH_{n-1} > D_{n-1} \\ Q_{n-L}, & n > L \ \wedge \ OH_{n-1} \leqslant D_{n-1} \end{cases} \tag{9}$$

Shortage inventory SI_n is the step function of the difference between D_n and OH_n, or

$$SI_n = [OH_n - D_n]^- = \begin{cases} D_n - OH_n, & D_n > OH_n \\ 0, & DF_n \leqslant OH_n \end{cases} \tag{10}$$

Inventory position IP_n is the sum of OH_n and IT_n, or

$$IP_n = OH_n + IT_n = OH_n + IT'_n + Q_n \tag{11}$$

where IT_n and IT'_n are defined the same as above.