PERGAMON

Computers and Electrical Engineering 28 (2002) 391–416

Computers and Electrical Engineering

www.elsevier.com/locate/compeleceng

Observability and state estimation of electrical power networks (sinusoidal signals)

Pierre Tréhin, Nicolas Héraud *

UFR Sciences et Techniques, SPE, Université de Corse, CNRS UMR 5134, BP 52, Quartier Grossetti, 20250 Corte, France

Received 20 July 1998; accepted 14 August 2000

Abstract

The aim of this article is to extract the totality of determinable parameters (measured or deduced) of an electrical power network in stationary state, in view to estimate their true values. These systems, different from conservation-law systems, are described by linear and bilinear equations which involve the complex intensities I_r and I_c, the impedances Z_r and Z_c, and voltages across the associated branch U_r and U_c. An algorithm allowing one to determine those determinable parameters is presented; this algorithm is based on the number of measured and unmeasured variables, on their localization and on their interdependence. Close to this observability study, a method of state estimation applied to the observable sub-system is presented. This method allows, besides the true value estimation, to detect and localize possible defects on the instrumentation, by the analysis of normalized residues. Finally, these algorithms will be applied to an electrical system.

Keywords: Observability; State estimation; Sensor positioning; Instrumentation

1. Introduction

Electrical networks are used in all industries, and it is sometimes necessary to know the state of the different branches of such a network, planning to control a process (study of the control system) or in view to insure the correct working order of the piloted process (analysis and/or supervision of the power network). We are going therefore to be brought to, on the one hand, determine all of the observable variables (measured or deduced) of a given network, then on the other hand to estimate true values of these variables from stemming collector values of measure.

* Corresponding author. Tel.: +33-49-561-0212; fax: +33-49-545-0162.
E-mail address: heraud@univ-corse.fr (N. Héraud).

0045-7906/02/$ - see front matter
PII: S0045-7906(00)00058-6

Control networks are the most often of continuous signal networks; the study of a such type of system has already been led [10]. On the other hand, the case of sinusoidal signal networks, more general, had not again been approached. We intend in this article to develop a method allowing to determine the observable sub-network, then to estimate the true state of this sub-network.

An electrical variable is described by, either the knowledge of its modulus and phase, or the knowledge of its reactive and active parameters. We will suppose in this article that we are in this last case.

A complex variable will be noted $\underline{X} = X_{\mathrm{r}} + \mathrm{j}X_{\mathrm{c}}$.

An electrical system comprising n nodes, v branches and m meshes is described by parameters of voltage $\underline{U}_i$ (voltage across a branch) and $\underline{E}_i = \underline{U}_i$ (voltage generator), component impedance $\underline{Z}_i$ and current $\underline{I}_i$; these variables are governed by the following general equations:

1.1. Kirchhoff laws

$$\text{Node statement} \quad \sum_{\text{node}} \underline{I}_i^* = \underline{0} \tag{1}$$

where the indication $*$ represents the true value of the parameter. This statement provides two linear equations:

$$\sum_{\text{node}} I_{\mathrm{r}i}^* = 0 \tag{2}$$

$$\sum_{\text{node}} I_{\mathrm{c}i}^* = 0 \tag{3}$$

$$\text{Mesh statement} \quad \sum_{\text{net}} \begin{pmatrix} \underline{U}_i^* \\ \underline{E}_i^* \end{pmatrix} = \underline{0} \tag{4}$$

This statement provides two linear equations:

$$\sum_{\text{mesh}} \begin{pmatrix} U_{\mathrm{r}i}^* \\ E_{\mathrm{r}i}^* \end{pmatrix} = 0 \tag{5}$$

$$\sum_{\text{mesh}} \begin{pmatrix} U_{\mathrm{c}i}^* \\ E_{\mathrm{c}i}^* \end{pmatrix} = 0 \tag{6}$$

1.2. Ohm law

The relationship voltage/current is expressed as

$$\underline{U} = \underline{Z} * \underline{I} \tag{7}$$

This relationship provides two bilinear equations:

$$U_{\mathrm{r}i} = Z_{\mathrm{r}i} I_{\mathrm{r}i} - Z_{\mathrm{c}i} I_{\mathrm{c}i} \tag{8}$$

$$U_{\mathrm{c}i} = Z_{\mathrm{c}i} I_{\mathrm{r}i} + Z_{\mathrm{r}i} I_{\mathrm{c}i} \tag{9}$$