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Static-dynamic uncertainty strategy for a single-item stochastic inventory control problem

Ulaş Özen [a,*], Mustafa K. Doğru [b], S. Armagan Tarim [c]

[a] Alcatel-Lucent Bell Labs, Blanchardstown Industrial Park, Dublin 15, Ireland
[b] Alcatel-Lucent Bell Labs, 600 Mountain Avenue, Murray Hill, NJ 07974, USA
[c] Department of Management, Hacettepe University, 06800 Ankara, Turkey

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ABSTRACT

We consider a single-stage inventory system facing non-stationary stochastic demand of the customers in a finite planning horizon. Motivated by the practice, the replenishment times need to be determined and frozen once and for all at the beginning of the horizon while decisions on the exact replenishment quantities can be deferred until the replenishment time. This operating scheme is refereed to as a "static-dynamic uncertainty" strategy in the literature [3]. We consider dynamic fixed-ordering and linear end-of-period holding costs, as well as dynamic penalty costs, or service levels. We prove that the optimal ordering policy is a base stock policy for both penalty cost and service level constrained models. Since an exponential exhaustive search based on dynamic programming yields the optimal ordering periods and the associated base stock levels, it is not possible to compute the optimal policy parameters for longer planning horizons. Thus, we develop two heuristics. Numerical experiments show that both heuristics perform well in terms of solution quality and scale-up efficiently; hence, any practically relevant large instance can be solved in reasonable time. Finally, we discuss how our results and heuristics can be extended to handle capacity limitations and minimum order quantity considerations.

1. Introduction and literature review

In inventory planning, freezing schedules for the timing of deliveries in advance, taking account of future uncertainties, is of practical interest. This need to fix the times of deliveries in advance, while allowing reasonable flexibility in the order size, has been at the heart of many industrial problems, such as coordinating suppliers and buyers in supply chain partnerships [5]; managing joint replenishments for multiple items [14] and planning for shipment consolidation in logistics [12]; master planning and leveling workload in Advanced Planning Systems [14]; buying raw materials on fluctuating price markets in purchasing [9]; etc.

Freezing delivery schedules alleviates the supplier–buyer coordination in supply chains susceptible to system nervousness, which arises when a formerly fixed order request for a certain period is replanned later. The deviations causing nervousness may be in the form of quantity adjustments and/or changes in delivery timing requests. Inderfurth [5] notes that nervousness due to deviations in delivery timing requests is considered as the most serious in practice and is referred to as setup instability. Blackburn et al. [1] encourage deliveries in periods where they are scheduled previously for dealing with the problem of nervousness.

Silver et al. [14, pp. 236–237] point out that freezing schedules is particularly appealing when items are ordered from the same supplier or require resource sharing. In such a case, all items in a coordinated group can be given the same replenishment period. This also allows a reasonable prediction of the level of the workload on the staff involved and is particularly suitable for advanced planning environments.

Shipment consolidation in logistics management is another area that benefits from scheduled deliveries. Mutlu et al. [12] describe shipment consolidation as the "practice of combining small size shipments into a larger load with the aim of benefiting from scale economies associated with transportation costs". The shipment consolidation policy using scheduled deliveries is called the "time-based policy" and noted to be popular in practice. In this policy, arriving orders are combined to form a large load, and consolidated shipments are released at periodic intervals. It is emphasized that this policy is important in terms of delivery reliability since it allows logistics providers to quote a delivery time.

Li et al. [10] discuss that, in many logistics systems, inventory replenishment by retailers involves a delivery request in advance without making a firm commitment on order quantity and give examples from industry. They argue that in contrast to more traditional approaches by which delivery request and quantity

* Corresponding author. Tel.: +353 18864448.
E-mail addresses: ulas.ozen@alcatel-lucent.com (U. Özen), mustafa.dogru@alcatel-lucent.com (M.K. Doğru), armagan.tarim@hacettepe.edu.tr (S. Armagan Tarim).

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decisions must be made at the same time, this approach allows retailers to postpone their quantity decision until better demand information is available, and, as a result, the mismatch between supply and demand is reduced.

In this paper, we consider the inventory problem faced by a manager who replenishes stock using the following practice. At the beginning of the planning horizon, he decides on the number and exact timing of delivery requests once and for all. This decision constitutes the "static" part of the manager's "static-dynamic uncertainty" strategy, the name coined by Bookbinder and Tan [3]. Each delivery request incurs a fixed cost. The exact order quantities for deliveries are determined only after observing the realized demands until that time. This decision constitutes the "dynamic" part of the strategy. The demand process in this paper is assumed to be the only source of uncertainty and follow a non-stationary pattern over a finite planning horizon. Moreover, static-dynamic uncertainty strategy considers a non-stationary ordering policy to better respond to the non-stationary nature of the demand. See [21] for a recent study on the cost of using stationary ordering policies in a standard inventory system with non-stationary demand and setup costs.

The static-dynamic uncertainty strategy is described first by Bookbinder and Tan [3]. They assume non-stationary demand and suggest a two-step heuristic solution method. In the first step, future replenishment periods are fixed at the beginning of the planning horizon using a Wagner–Whitin type model. In the second step, subsequent order quantities are determined on the basis of demands that have become known at a later point in time. Proportional end-of-period inventory holding and fixed-ordering costs are taken into account. Instead of adopting a penalty cost approach, service level constraints are imposed in each period. In a related work, Bookbinder and H'Ng [2] introduce and test a rolling horizon framework. Tarim and Kingsman [17] address the same problem and provide a mixed integer programming (MIP) model to simultaneously answer the questions on the exact timing of future replenishments and corresponding order quantities. Tarim et al. [16] mainly focus on the computational issues and provide an efficient computation approach to solve the MIP model in [17]. Tempelmeier [19] addresses the same problem under a fill rate constraint as a service measure. Finally, Tarim and Kingsman [18] relax the service level constraints and present an approximate model for the penalty cost case with normally distributed demands. In all the aforementioned works the inventory control policy is in the form of static-dynamic uncertainty strategy. Although the demand process is assumed to be non-stationary and stochastic, all these papers formulate certainty equivalent mathematical programs and analyze the resulting deterministic problems.

In a closely related inventory control problem, Sox [15] investigates the stochastic demand and dynamic costs case. The control strategy adopted can be classified as "static uncertainty", in which replenishment periods and order quantities are all fixed at the beginning of the planning horizon. A solution algorithm that resembles the Wagner–Whitin algorithm is proposed. In a similar work, Vargas [22] addresses the stochastic version of the Wagner–Whitin problem, employing again the static uncertainty strategy. Tempelmeier [20] studies a multi-item application of static uncertainty strategy under capacity constraint and fill rate service measure, and develops a column generation heuristic. A static uncertainty strategy is useful for supplier–buyer coordination and advanced planning purposes; however, the lack of any control over unfolding uncertainties can be a downside.

Our work is also related to the literature on "periodic (cyclical) demand" models[1] in which independent and non-stationary demands that follow a periodic pattern are studied; hence, we review some of the main results. Karlin [6,7] is the first to study a periodic demand model and to show that a base stock policy is optimal. Zipkin [24] extends Karlin's results under average cost criterion. Morton and Pentico [11] provide upper and lower bounds for the optimal base stock levels. Finally, Sethi and Cheng [13] consider fixed cost of ordering and Markov modulated demand, and prove that the optimal policy is (s,S) type. All these papers adopt a "dynamic uncertainty" strategy in which both replenishment times and quantities can be decided dynamically through the course of the planning horizon. Hence, they are distinctively different from our work.

Our contribution in this paper is multi-fold. We provide a model and a solution algorithm for finding an optimal solution for the static-dynamic uncertainty strategy under penalty cost or service level constraints. In contrast to the related literature presented above, in which certainty equivalent mathematical programming formulations dominate, a dynamic programming (DP) based approach is adopted. The computational complexity of this algorithm is non-polynomial. Although it yields, for small and medium size instances, the optimal solution in reasonable time, for large instances it is infeasible. Therefore, we develop two heuristics: Approximation Heuristic (AH) and Relaxation Heuristic (RH). While AH approximates the true cost-to-go function, RH relaxes a constraint in the original problem. Our numerical experiments show that both heuristics perform well in terms of solution quality and computation time, and find the optimal policy over a wide range of model parameters. Any practical size instance can be solved to near-optimality using these heuristics. Finally, we discuss how our results and heuristics can be extended to handle capacity limitations and minimum order quantity considerations.

The remainder of the paper is organized as follows. In Section 2, we introduce the notation and the model for the penalty cost case. Next in Section 3, we characterize the optimal policy, provide bounds for the optimal policy parameters and costs (Section 3.1), and discuss how to compute these parameters (Section 3.2). We develop two heuristics, Approximation Heuristic and Relaxation Heuristic, which are discussed in Sections 4 and 5. In Section 6, we introduce the model and the heuristics for the service level case. After the numerical results in Section 7, we present in Section 8 the analysis of a model that incorporates extensions such as capacity limitations and minimum order quantities. Finally, we conclude in Section 9 with directions for future work. The proofs are given in an Online Appendix.

2. Model and notation

Consider a single-stage inventory system in a finite horizon under periodic review. Without loss of generality, period length is assumed to be one time unit, and $t \in \{1,2,\ldots,N\}$ is the index for periods with N being the last period in the planning horizon. Let D_t be a nonnegative continuous random variable denoting the demand in period t. Even though demands in different periods are assumed to be independent, they are not necessarily identically distributed. Any unfulfilled demand is backlogged and a unit penalty cost of b_t per unit is incurred at the end of period t. Similarly, for inventory carrying, holding cost h_t per unit is assumed for period t. The system is replenished from a supplier with ample stock and without loss of generality, we assume replenishment lead time to be zero.[2] A fixed cost A_t is incurred if a

[1] Demands in different periods have different distributions, but they repeat in every cycle of K periods.

[2] The model can be extended to include non-zero fixed lead times since cost functions can be rewritten in terms of inventory position as oppose to net inventory.