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The order and volume fill rates in inventory control systems

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A R T I C L E   I N F O

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A B S T R A C T

This paper differentiates between an order (line) fill rate and a volume fill rate and specifies their performance for different inventory control systems. When the focus is on filling complete customer orders rather than total demanded quantity the order fill rate would be the preferred service level measure. The main result shows how the order and volume fill rates are related in magnitude. Earlier results derived for a single-item, single-stage, continuous review inventory system with backordering and constant lead times controlled by a base-stock policy are extended in different directions. Demand is initially assumed to be generated by a compound renewal process. An important generalization then concerns the class of customer order-size distributions, i.e. compounding distributions, with increasing failure rate for which the volume fill rate *always* exceeds the order fill rate. Other extensions consider more general inventory control review policies with backordering, as well as some relations between service measures. A particularly important result in the paper concerns an alternative service measure, the customer order fill rate, and shows how this measure always exceeds the other two more well-known service measures, viz. the order fill rate and the volume fill rate.

1. Introduction

Service levels are used in inventory control systems for performance evaluation and in target setting as substitutes for shortage costs that are hard to estimate. An early but good review of standard service level measures and their relationships to shortage costs and different inventory control policies is provided by Schneider (1981). Discussions on service measures are also found in most contemporary textbooks on operations management. One of the most commonly used performance measures in inventory control is the *fill rate* (FR), defined as the fraction of demand that can be met, in the long-run, immediately from inventory without shortages (Silver et al., 1998, p. 245). Somewhat less common as a performance measure is the *ready rate* (RR), specified as the fraction of time during which the on-hand stock is positive.

For pure Poisson demand, as well as for continuous, normally distributed demand, it is well known that the FR and the RR are equivalent measures (Silver et al., 1998, p. 245; Axsäter, 2006, pp. 57–61). In the case of pure Poisson demand this follows immediately from the PASTA property and from the fact that each customer only orders one unit at a time. According to the *PASTA* property of Poisson processes the fraction of customer arrivals, i.e. demands that find the stock at a certain level is equal to the fraction of time that the stock is at this level (Wolff, 1982). Because the normal distribution can only be an approximation of the true demand distribution, the result holds only approximately in this case.

However, this result does not hold for general demand processes. Consider, for example, the case when a low, but positive, on-hand stock during most of the time results in a fairly high RR. The FR might nevertheless be low due to a few large customer orders that cannot be met immediately from the available low on-hand stock. This suggests that if a compound element, specified by a positive random variable, is added to the Poisson process, that is, if customers can demand more than one unit at a time, then the relevance of the RR is less obvious. Furthermore, if one also deviates from the assumption that customer orders arrive randomly over time, as specified by a Poisson process, then the RR performance measure becomes even more dubious. If, for example, demand is highly seasonal, it primarily makes sense to keep stock in anticipation of the season, but less so at other times during the demand cycle. Other types of regularities with respect to demand occurrences can induce similar effects. To be able to model such regularities, a more general demand arrival process, a renewal process (Tijms, 2003) can be used. Consequently, the demand process can be modeled as a compound renewal process. This includes the compound Poisson process and the pure Poisson process as special cases.

Even if the RR may not, in general, be a valid service measure, the focus on complete order fulfillment is of course highly

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relevant in many settings. Hausman (1969) contains an approximate treatment of this issue in a cost optimization setting. In our paper, we consider an exact specification of the *order* (line) fill rate (OFR) and contrast it to the standard fill rate for which we henceforth use the specification *volume* fill rate (VFR). The OFR is the fraction of customer orders (irrespective of their size) that can be met in full immediately from inventory without shortages, whereas the VFR is simply the standard FR, as defined above.

The OFR with its focus on order completion is closely related to the (*Order*) *Fill Rate* metric defined in the *SCOR*-model as the percentage of (complete) orders shipped from inventory within a certain time frame, say 24 h. For a brief description of *SCOR* see for instance Simchi-Levi et al. (2008, pp. 381–382). Other related service measures in the *SCOR*-model are the *On-Time-In-Full* metric and the *Perfect Order Fulfillment* metric. The latter two metrics are also related to the amount of orders delivered in the quantities requested, but their scope is broader in the sense that they also consider customer orders including several different SKUs or order lines. In this sense these two metrics are related to the compound service measures considered in Song (1998) and Hausman et al. (1998).

The main results in this paper are the following. First, if the customer order-size distribution, i.e. the compounding distribution in a renewal demand process, exhibits increasing failure rate, then it is shown that the VFR *always* exceeds the OFR. The opposite is true for a decreasing failure rate. For a constant failure rate the two service measures are always equal. The only discrete distribution with a constant failure rate is the geometric. This result generalizes considerably an earlier result in Larsen and Thorstenson (2008). Their results are also extended here to encompass more general inventory control policies which may involve lot sizing. Second, the current paper defines an alternative service measure, the customer-order fill rate (CFR). This measure generalizes a service measure definition given in Chen et al. (2003) and in Thomas (2005). Our paper establishes that for given inventory system control parameters the CFR always shows a higher service performance than the other two more well-known fill-rate service measures, the OFR and the VFR.

The rest of the paper is organized as follows. In Section 2 we reiterate, for the sake of completeness, basic results regarding the relation between the OFR and the VFR for the continuous review, base-stock system with backordering and constant lead times. Then, in Section 3 the assumptions about the customer order-size distribution are generalized, so that the results hold for much broader classes of distributions. In Sections 4 and 5 a general continuous review control policy assuming backlogging of unfilled demand is examined. In Section 4 it is shown that the results of Sections 2 and 3 can be generalized to this control policy and this result is further exemplified for the case of a continuous review (r, nq) policy under a compound Poisson demand process. In Section 5 the CFR service measure is introduced and its relationships to the OFR and the VFR are derived. Finally, Section 6 contains the conclusions.

2. Basic assumptions and specifications

The following basic assumptions about the inventory system to be controlled are stated in Larsen and Thorstenson (2008). It is assumed that demand is driven by a compound renewal process with inter-arrival times represented by non-negative and continuous i.i.d. random variables. The customer order sizes at the demand instances follow i.i.d. discrete random variables defined on the positive integers. The latter assumption implies that a customer always orders at least one unit. This assumption can be imposed without loss of generality, because the frequency of customer orders can be adjusted independently through the intensity of the renewal process. The term *delayed distribution* is often used to indicate that the support for the customer order-size distribution has 1 (one) as its lowest value. In the following it is understood that each time we refer to a distribution (geometric, logarithmic etc.) it is its delayed distribution.

Moreover, the inventory is controlled by a standard base-stock policy and demands not satisfied immediately from inventory are backordered. Partial deliveries are used for customer orders that cannot be satisfied in full directly from the inventory. The inventory level is monitored continuously and a replenishment order is issued immediately, whenever the inventory position (net inventory level + replenishment orders outstanding) is below the base-stock level (order-up-to level) S. There is no lead-time uncertainty. Note, that the base-stock policy is not the optimal control policy in general for renewal demand processes. However, under a standard cost structure it is the optimal policy, if the decision epochs for replenishments are restricted to the points in time immediately after a demand occurrence. Without this restriction it is also optimal in the special case of a compound Poisson process. Furthermore, it is a simple policy and it is often used for purposes of benchmarking.

Now, let the customer order sizes be distributed as the positive integer-valued stochastic variable J. Furthermore, let the aggregate demand during the constant time interval of length L be described by the integer stochastic variable D_L. This time interval is either immediately preceded or followed by a demand instance and L is also the length of the constant lead time. Obviously, the distribution of D_L depends on the distribution of J, as well as on the renewal process describing the demand instants.

The OFR service measure is specified as the probability that an arbitrary customer order can be satisfied completely from inventory without delay. Under the assumptions stated, when a customer order arrives, with probability $P(D_L = k)$ there will at the time of the arrival be a net inventory of $S-k$. With probability $P(J \le S-k)$ the whole customer order can then be satisfied immediately. Hence, the order fill rate, OFR, for a given base-stock level S is obtained as

$$OFR(S) = \sum_{k=0}^{S-1} P(D_L = k)P(J \le S-k) \tag{1}$$

The VFR service measure is specified as the probability that an arbitrary demand unit can be satisfied from inventory without delay. It can be derived by applying a similar reasoning, see Larsen and Thorstenson (2008), as in the case of the OFR and can then be expressed as

$$VFR(S) = \frac{\sum_{k=0}^{S-1} P(D_L = k)\left[\sum_{j=1}^{S-k} (j-S+k)P(J=j) + S-k\right]}{E[J]} \tag{2}$$

Both the OFR and the VFR are easy to update iteratively. In particular,

$$OFR(S+1) = OFR(S) + \sum_{j=0}^{S} P(D_L = S-j)P(J = j+1), \quad S = 0,1,\ldots \tag{3}$$

and

$$VFR(S+1) = VFR(S) + \frac{1}{E[J]}\sum_{j=0}^{S} P(D_L = S-j)[1-P(J \le j)], \quad S = 0,1,\ldots \tag{4}$$

where $OFR(0) = VFR(0) = 0$.

The next basic assumption specifies the distribution for the customer order sizes. It is assumed that the stochastic variable J that represents the customer order size follows a Negative