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A characteristics-based finite element method for transmission line problem

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ABSTRACT

This paper presents a numerical model of transmission line equations based on a combination of the finite element method and the generalized method of characteristics. A local system of the transmission line finite element is obtained using the generalized method of characteristics applied to the Telegrapher's equations. In this way, a spatial functional approximation using local shape functions together with the generalized trapezoidal rule used for time integration as it is done in the classical finite element formulation is avoided and higher accuracy of results is obtained. In order to show the essential principles of the proposed numerical method, for sake of simplicity, the scope of the paper is restricted to a single transmission line problem at the low frequency regime.

1. Introduction

Several numerical methods both in time and frequency domains can be used for transmission line (TL) analysis. A TL problem in time domain can be solved using π-circuits [1], the modal method [2,3], the finite difference method (FDTD) [2,4], and the method of characteristics [5]. Furthermore, the generalized method of characteristics (GMC) has been successfully applied to lossy TL problem [6,7]. The finite element time domain (FETD) method for a TL problem has been recently developed and applied to linear, non-linear and multi-conductor problem [8–11]. This method exhibited very good numerical features in TL analysis although some authors earlier had expressed unsuitability of the finite element method for Telegraphers' equations due its hyperbolic nature, as reported in [12]. The finite element method (FEM) has also been successfully applied to the Telegrapher's equations in frequency domain [13], where it also showed very good numerical features.

Summed up briefly, the finite element (FE) numerical model of a TL problem substitutes partial differential equations (PDEs) with algebraic equations expressed in the terms of appropriate nodal variables. Formulation of a FE local system consists of expressing a functional dependence of TL and/or lumped element, which are given in terms of branch variables, as the corresponding functional dependence in terms of nodal variables. The FE model of TL is based on the weighted residual method applied to the Telegrapher's equations to obtain ordinary differential equations (ODEs) which are further subjected to the appropriate time integration scheme.

Generally, there are two possible formulations of the FEM model for a TL problem. First, one can model Telegrapher's equations given by the second order PDE using the weighted residual method, for space and the Newmark method, used in FE analysis to model dynamic systems [14,15], for time integration. This approach requires three local shape functions, i.e., it needs three nodes per one-dimensional FE.

The second formulation of the problem is based on the weighted residual method applied to a system of two Telegrapher's equations of the first order and the generalized trapezoidal rule for time integration as presented in [9].

Antecedent formulation yielded better numerical features for transient analysis because less numerical oscillations appeared when rapid changes of functions occurred. To conclude, our experience showed that the FE model, based on Telegrapher's equations given in the form of two coupled first order PDEs, is more convenient when modeling a TL problem then to model one PDE of the second order.

On the other hand, the GMC is claimed to be the best method for hyperbolic PDEs and is widely used for numerical calculations of transient wave processes not only in electromagnetic transients problem on electric TLs but also for fluid transients calculation in pipe networks. However, great difficulties are encountered when applying this method to a branched pipe network analysis because of the limiting Courant's criterion for the time integration [16] due to irregular discretization where FEs can significantly vary in size. Improvement of the FEM applied to fluid transients in pipe network using the GMC is accomplished in [16]. Since different areas of engineering have similar problems and consequently similar or same methods can be used, this paper presents a possibility of applying the GMC to the FE formulation of TL problem by generating a FE matrix and vector according to the GMC in order to improve FEM

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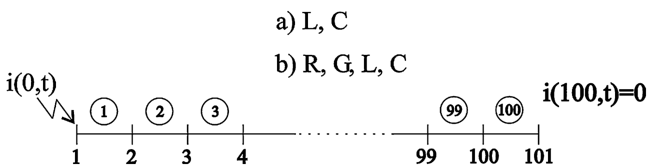

Fig. 1. The FE mesh and boundary conditions of the (a) lossless and (b) lossy conductor.

to TL problems. Analysis of transients on multi-conductor TL line with frequency-independent and frequency-dependent parameters using the proposed approach will be the goal of future research.

2. Classical FEM procedure

Since the fundamentals of the FEM in the time domain for a TL problem have been already described in [9], here only a brief notice on approximation of an operator using the weighted residual method will be given.

Various choices of trial (shape) and testing (weighting) functions used in the weighted residual method approach allow for various possibilities of operator approximation. For TL problem it was determined that a constant weighting function equal to a unit value and linear shape functions on each FE yield the best approximation. Advantage of the FEM over the other methods is manifested in easy handling with complex network with distributed and/or lumped parameters due to the assembling procedure, which is inherent to the FEM. Using linear spatial interpolation (shape) functions for TL FEs and constitutive equations for lumped elements together with time integration performed by the generalized trapezoidal rule, the local system of algebraic equations for every FE can be obtained.

2.1. Transmission line FE

Propagation of disturbance on a two-conductor lossy TL is governed by the Telegrapher's equations in terms of voltage and current.

$$\begin{aligned} -\frac{\partial u}{\partial x} &= Ri + L\frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial x} &= Gu + C\frac{\partial u}{\partial t} \end{aligned} \tag{1}$$

where R, L, C and G are the resistance, inductance, capacitance and conductance per unit length, respectively. Using the weighted residual method and time integration performed by the generalized trapezoidal rule also known as ϑ-method a local system of FE equations can be obtained [9].

2.2. FE of lumped circuit elements

Besides FEs with distributed parameters, electrical networks can usually have lumped elements, i.e., resistors, capacitors and inductors. The FE formulation of such elements has already been derived in [9].

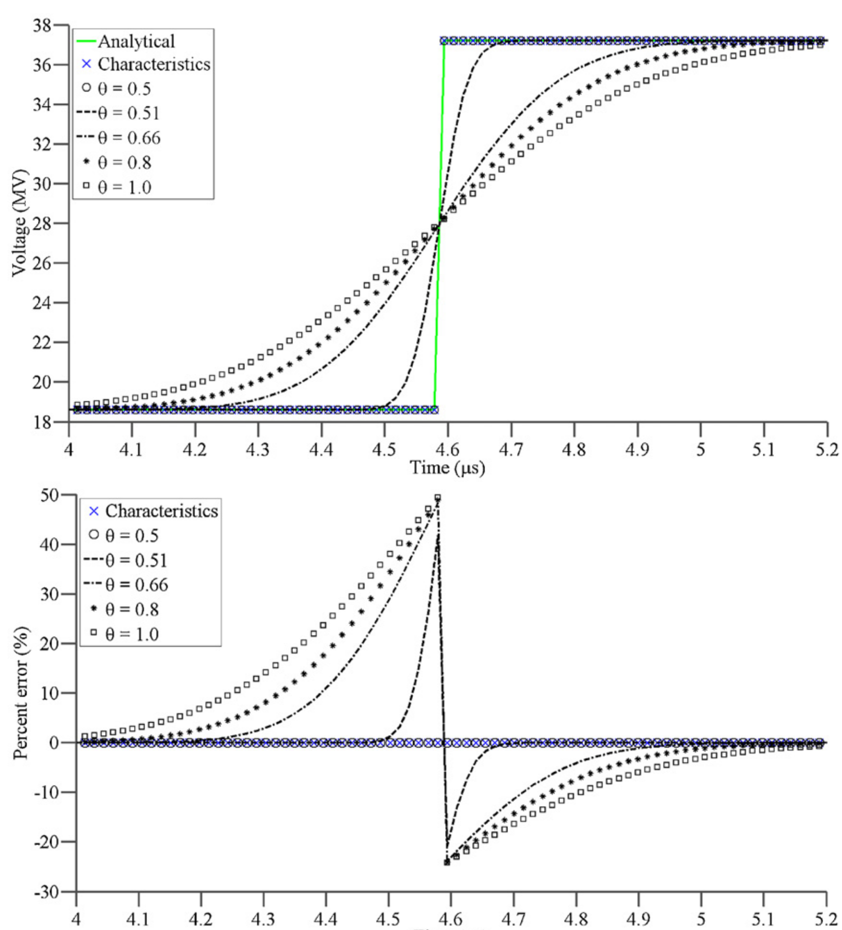

Fig. 2. Voltage wave front and percent error of voltage wave front at $x = 25$ m.