Electric Power Systems Research 79 (2009) 669–679

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Backward solution of PV nodes in radial distribution networks

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ARTICLE INFO

Article history:
Received 12 June 2007
Received in revised form 13 June 2008
Accepted 6 October 2008
Available online 2 December 2008

Keywords:
Radial distribution networks
Backward/forward method

ABSTRACT

In this paper an iterative backward methodology to solve radial distribution networks with fixed voltage (PV) nodes and with constant power loads or mixed loads (with at least one component with constant power) is proposed. The method developed, although deriving conceptually from the backward/forward (b/f) methodology, presents only the backward phase in which all the network variables are evaluated.

In the methods developed up until now for the solution of such systems, PV nodes are taken into account at the end of each iteration by evaluating, based on the known quantities of the network, the unknowns associated with PV nodes. In the methodology developed here the unknowns relevant to PV nodes are considered within the search process together with the unknown state variables. The proposed method at each iteration requires the solution of a network made up only of impedances; for such a system, supplied only at one node, the susceptances of the PV nodes are unknown as well as the currents in shunt impedances of the terminal buses. In order to solve such a system, a simple and efficient technique has been established. It allows the determination during the backward sweep of all the unknowns. The main and most important feature of the simulation of PV nodes with shunt reactance is the high precision of results related to reactive power injection at PV nodes. The applications indeed show that precision does not differ from that related to the use of the classical Newton–Raphson method; furthermore, also the number of iteration is similar with reduced CPU times. After having reported the models of PV nodes already existing in the literature in the field of b/f analysis methods, the general methodology for solving a radial network made up of impedances is briefly presented. The new analysis method and its implementation are then presented in detail. The results of the applications carried out show the good performance of the model in terms of both speed of convergence and, mainly, of precision.

1. Introduction

In some cases the classical Newton–Raphson (NR) method in radial distribution networks analysis presents difficulties. For this reason, specific methodologies have been developed for these kinds of electrical systems. Nevertheless, the NR approach, taking into account some modifications based on particular features of the radial structure, has been applied in [1], where load flow equations are written in terms of new variables resulting in a set of $3N$ equations for a network with $N+1$ buses. As $2N$ equations are linear and N are quadratic, a computational scheme, based on the NR method, is developed in order to increase the efficiency of the methodology.

For constant power loads or mixed loads, with a constant power component, the analysis method more frequently adopted in radial distribution systems is the iterative backward/forward (b/f) methodology. The latter shows three different variants: the current summation method, the power summation method and the admittance summation method.

For the current summation method, each iteration is articulated in two steps. In the first phase (backward sweep), on the basis of a known voltage profile, currents derived from loads and from capacitive admittances of the lines are evaluated and from these, starting from terminal branches and going towards the source node, the currents flowing in all the branches of the system are determined. In the second phase (forward sweep), starting from the source node and going towards the terminal nodes, the voltages at all nodes are calculated. Once the iteration is completed for each node, the starting and ending values of the voltage are compared. If the difference is below a fixed value for all the nodes, the iterative process stops. Otherwise, another iteration is performed by updating loads currents based on the bus voltages just calculated.

In the power summation method the procedure is similar but with the obvious difference that in the backward sweep the branch power flows are evaluated.

In the admittance summation method the loads are simulated in each iteration by means of admittances. In the backward sweep, starting from the terminal nodes, the driving admittances of each

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doi:10.1016/j.epsr.2008.10.002

node are determined; once the source node driving admittance is found, it is possible to evaluate the total current requested from the system. In this way, during the forward sweep and starting from the source node, bus voltages can be calculated. In the traditional b/f methodology, any is the load simulation, determination of the node voltages must be executed sequentially, that is, the voltage calculation of a bus needs the evaluation of the voltages of all upstream nodes.

If the network is meshed, the system can be turned into a radial system, executing a number of cuts in the branches (cut number is equal to the number of independent meshes) and introducing, for each cut, a new unknown which is the current to be injected between the two nodes created by the same cut so as to equal the voltage at the same two nodes. The calculation of this current is performed using the multi-port compensation method presented by Tinney [2].

The introduction to distribution systems of small, dispersed generation units, requiring the reactive power necessary to sustain the voltage at the same nodes, implies that it is necessary to set up distribution systems analysis methods where PV nodes can also be included. These nodes inject into the system a fixed value of active power at a given fixed voltage; the reactive power at these nodes is the unknown as well as the voltage displacement. The presence of PV nodes is considered in [3] within a power flow calculation method valid for slightly meshed distribution and transmission systems. At each iteration the PV node voltage is modified, giving it a fixed value and its displacement is equal to that calculated for the voltage at the same iteration. The reactive power at the same node that is necessary to keep the node voltage at the required value is calculated using the secant technique and using voltage values at the same node that have been calculated in the two preceding iterations. The methodology used to turn meshed into radial systems is also followed in [4] to consider PV nodes. A mesh is created between the reference node and the PV node, and then a voltage source with the same prefixed value for the PV node is inserted into this mesh. The mesh is opened through a cut and the reactive power, that must be injected to set to zero the voltage difference between the PV node and the voltage source terminal, is calculated. The reactive power is then updated at each iteration on the basis of a sensitivity matrix which, using the hypothesis that all nodes have voltage magnitudes close to 1 p.u. and negligible relative displacements, is the same as the reduced equivalent impedance Thévenin matrix. This methodology is modified in [5] in order to speed up the convergence. At the end of each iteration, and before the update of the current values, the nodal voltages are corrected considering the fact that in the network there are only compensation current variations, both in the cuts representing real meshes and in the cuts performed on fictitious meshes (PV nodes). A three-phase power flow, based on the methodology developed in [3], is presented in [6] to solve meshed distribution systems with dispersed generation (PV nodes), unbalanced loads, voltage regulators and shunt compensation banks. For the same type of network, the same methodology is applied in [7] to evaluate the currents due to different permanent fault conditions; a hybrid Thévenin equivalent impedance matrix links the compensation currents of the real meshes and those of the fictitious meshes related to the PV nodes and to the faults with the relevant voltage differences. In [8] the method presented in [5] is again considered, with some modifications in the calculation of the compensation currents of the PV nodes that are made in order to ensure that at each iteration the current phasor to be injected into the PV nodes is at 90° with the nodal voltage phasor. A different approach, but still belonging to the compensation currents method family, is presented in [9] where the calculation of the unknown quantities for the PV nodes is performed at the end of the backward phase on the basis of the distribution of currents in the branches upstream of the PV nodes. A suitable sequence in the solution method for the calculation of the currents in the real and fictitious (PV nodes) meshes, as well as the updated currents in the branches after the calculation of each of these currents, allows a reduction of errors and thus accelerates the end of the solution. In [10], for the fictitious meshes related to the PV nodes, the compensation currents are deduced by means of a sensitivity matrix and the voltage differences existing between the cut nodes. Whereas the voltage on the network side is known in phase displacement and in magnitude at the end of each iteration, the voltage on the ideal generator side is known only in magnitude, the latter being equal to the specified value for the PV node. The phase displacement of this voltage is deduced by imposing that the power injected into the system is only reactive power; thus, once the forward phase is finished, calculation of the displacements of the voltage phasors to be associated with the PV nodes ideal generators is performed, followed by calculation of all the compensation currents through the reduced equivalent Thèvenin impedance matrix. In [11], the authors develop a methodology to solve radial networks considering at each iteration the loads as impedances. In the backward phase, the currents circulating in the impedances derived from the terminal nodes are arbitrarily chosen, and then the voltage at the source node is calculated considering a passive network of impedances. The ratio between the voltage imposed at the source node and the one just calculated at the same node is the correction factor which allows the calculation of the actual values of bus voltages, load currents and branch currents. Then, for each PV node, the value of the reactance for which the bus voltage magnitude has a prefixed value is calculated.

In this paper, the general analysis methodology shown in [11] is again considered. While in [11], as in the rest of the cited references, the PV nodes are treated at the end of each iteration, in the methodology proposed here during the backward phase of each iteration the unknown susceptances associated with each PV node are considered together with the unknown currents in the shunt terminal impedances. The physical constraints that some nodes impose on voltages allow the determination, during the backward phase, of all the unknowns one by one. This fact, as is shown in the applications, can allow a great acceleration in identifying the solution. After a brief presentation of the general analysis methodology already set up by the authors in [11], the process through which all the unknowns of the system are determined, based on some defined node typologies, is outlined. Finally, the results of the applications are reported as well as their comparison with those attained using the classical Newton–Raphson method. Indeed, the most important feature of the PV model here presented is the precision of the results; due to the fact that the methods, which use compensation currents to simulate these kind of nodes, give approximate results, the most significant simulation method, for the comparison in terms of precision, is the NR one. Moreover, having the NR method optimal features in terms of speed of convergence, the comparison among the number of iterations can supply interesting indications about the performances of the model proposed.

2. The backward analysis methodology [11]

Consider the network shown in Fig. 1, composed of N four-pole (Γ type) networks connected in cascade, each having a series impedance $\boldsymbol{Z}_{ser,i}$ and a shunt impedance $\boldsymbol{Z}_{sh,i}$ ($i = 1, 2\ldots N$); the network is supplied by the voltage source V_{source} (hereafter, bold letters indicate phasors or complex quantities; normal letters real quantities).

For load-flow calculations, this network can be assumed to be a circuital model of a distribution line supplied at one end (at voltage