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A network-topology-based load flow for radial distribution networks with composite and exponential load

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ABSTRACT

A network-topology-based method to solve the load-flow problem of radial distribution networks is reported in this paper. The proposed technique, based on network graphical information allows power flow equations formulation in matrix form to satisfy the need of distribution automation. The technique only requires tabulation of the input information for line data in such way that the receiving end node must be in an ascending order. A directed graph, of a radial network represented by a *nodes-by-nodes* sparse matrix allows detection of the path of power flow from the reference node to the leaf end. Traversing the directed graph in depth-first search form, the power flow paths (downstream nodes for each node including the node itself) are detected. A *BN* connection matrix is constructed based on the discovered paths. The lengths of the discovered paths explore the number of downstream nodes from each node including the node itself. The bus-injection to branch-current (*BIBC*) is built by assigning unity to the nodes of the discovered paths. The proposed method (*PM*) also allows dynamic building of the two matrices: *BIBC* and branch-current to bus-voltage (*BCBV*) matrix, used to find out the load flow solution. Reconstruction of these matrices takes place automatically by just changing two elements in the sparse matrix *S* reflecting changes in network configuration. The *PM* is compared with the other methods to demonstrate its effectiveness. The convergence ability of method is also evaluated for different types of load-modeling, different tolerance values, loading conditions and *r*/*x* ratios. The results of the applications of the proposed methodology to a set of networks taken from the literature on the topic along with conclusion are presented.

1. Introduction

Distribution Automation Systems have evolved both in concept and implementation over a period of time. The distribution power flow has influenced other applications such as network optimization, *VAR* planning and switching. The distribution systems, characterized by their prevailing radial nature and high *r*/*x* ratio, render them to be ill-conditioned and make the traditional Newton-Raphson (*NR*) [1] and fast decoupled power flow (*FDPF*) [2] solution techniques unsuitable. Consequently many power flow algorithms specially suited for distribution systems have emerged and are well documented [1–16]. These methods are roughly viewed as node based and branch based methods. The first category has used node voltages or current injections as state variables and requires information on the derivatives of network equations. The *Z*-bus method [3], *NR* based algorithms [1,4,5] and *FDPF* based algorithms [2,6,7] have revolved around this group. The second category has adopted branch currents or branch powers as state variables and involved only basic circuit laws. The backward/forward (*b*/*f*) sweep based methods [8–14] and loop impedance [15] based methods have found themselves in this group. However, the formulation and the algorithm are different from *NR* technique, rendering this category to be unsuitable for other applications such as optimal power flow, state estimation, etc. for which the former seems to be more appropriate. A fast decoupled distribution power flow (*FDDPF*) based on equivalent line current flows, which are rotated by appropriate line admittance angle for decoupling the problem, has been suggested [7]. A compensation based technique that exploits radial structure to achieve high speed, robust convergence and low memory requirement, for weakly meshed distribution systems has been explained [8]. A simple and efficient branch-to-node matrix based power flow (*BNPF*) for radial distribution systems has been presented [9]. This method is based on the formation of a constant sparse upper triangular branch-to-node matrix. However node voltages are calculated from branch voltages increasing number of equations to be solved. In spite of the fact that a number of distribution power flows are available, a generalized strategy is yet to be developed. There is therefore a significant need for developing a specific fast power flow algorithm exclusively for distribution systems. Recent research proposed some new ideas on how to deal

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with the special topological characteristics of distribution systems [9,17]. A simple technique is used to sort input data ([*SD*] matrix) and based on network graphical information (node-load to branch-load [*NLBL*] matrix) power flow equations are formulated in matrix form to satisfy the need of distribution automation [18]. In the algorithm input data are arranged to obtain sorted power injections, losses, branch currents and complex voltages using the novel equations.

By using topological characteristics of distribution network, the bus injection to branch current matrix (*BIBC*) and the branch current to bus voltage matrix (*BCBV*) and a simple multiplication are used to obtain the load flow problem solutions technique [17,19]. Same has been applied for two different topologies under distribution reconfiguration [20] for loss minimization. The network topology based algorithm as in [17] using current as variable has problems for large radial networks, like developing the two constant matrices (*BIBC*) and (*BCBV*) by writing special program. It also takes more computation time to converge the solution, when compared with the *b*/*f* technique and ladder network theory methods as expressed in [18] for the given data as in [21]. Load flow solution in [22] is obtained for radial distribution system using network topology to build two matrices. 'Bus injection to Node power matrix' is used to identify the sum of all active and reactive powers connected to nodes beyond a particular node and 'Line loss to node power matrix' is used to calculate the power loss in all the lines connected beyond the node under consideration. A simple algebraic expression of voltage magnitude is used. A new and accurate method for load-flow solution of radial distribution networks with minimum data preparation is reported [23,24]. The node and branch numbering need not to be sequential like other available methods. The proposed method does not need sending-node, receiving-node and branch numbers if these are sequential. The proposed method uses simple equation to compute the voltage magnitude and has the capability to handle composite load modeling. A reliable power flow technique based on equivalent node current injections is presented [25], which are computed from the specified load powers, and line voltage drops for radial distribution systems. The equations of node current injections are solved iteratively for line voltage drops. The approach exploits the features of both node-based and branch-based distribution power flow approaches. It is simple and uses a sparse constant jacobian matrix that needs to be factorized only once in the iterative process. A new general *b*/*f* procedure showing some interesting features that improve its performance in terms of convergence speed and calculation effort is presented [26]. The features that fundamentally are responsible for such improvements concern the main steps of the *b*/*f* procedure. The starting voltage profile solution is different from the flat profile and is suitably modified. In the backward phase and starting from the second iteration, the branch currents variations due to the loads changes are evaluated. The latter variations are calculated on the basis of the difference of nodal voltages at beginning and at the end the iteration. Finally, the adopted convergence criterion is based on the entity of the difference between each load node current in two subsequent iterations. The convergence ability of distribution systems power flow algorithms, which are widely used for distribution systems analysis, is compared with different voltage-dependent load models [27]. The convergence ability of methods are also evaluated for different tolerance values, voltage levels, loading conditions and *r*/*x* ratios, under the wide range exponents of loads. Results show that Ratio-Flow method is preferable from other methods. Reference [17] shows that the ladder network theory [28] and *b*/*f* substitution [29] are more flexible for the changes in data as compared to the other methods: Modified Gauss-Seidel Algorithm [30] and Network topology based method [31]. If any modification is to be made in the system data, then it is very easy in the ladder network theory and forward–backward substitution methods. But the same thing is difficult with the network topology based method, implicit 2-bus method and modified Gauss-Seidel method since the whole matrices need to be updated.

In the algorithm proposed in this paper, improvements in the implementation are executed, and few control actions are incorporated into the formulation. In the *PM*, the only input data required is the conventional bus-branch oriented data used by most utilities. A directed graph, of a radial network represented by a *nodes-by-nodes* sparse matrix allows detection of the path of power flow from the reference node to the leaf end. Traversing the directed graph in depth-first search form, the power flow paths (downstream nodes for each node including the node itself) are detected. A *BN* connection matrix is constructed based on the discovered paths. The lengths of the discovered paths explore the number of downstream nodes from each node including the node itself. The bus-injection to branch-current (*BIBC*) is built by assigning unity to the nodes of the discovered paths. The proposed method (*PM*) also allows dynamic building of the two matrices: *BIBC* and branch-current to bus-voltage (*BCBV*) matrix, used to find out the load flow solution. Reconstruction of these matrices takes place automatically by just changing two elements in the sparse matrix *S* reflecting changes in network configuration [32], thus avoiding updating the whole matrices. The method is then applied for load flow studies, aiming to reduce the computational time associated with. This technique compares favorably in terms of speed, convergence and computer storage requirement to other methods reported by technical literature. The detailed load-flow results for eight test systems with different kinds of load-modeling (constant power load, constant current load, constant impedance load, composite load and exponential load), different tolerance values, loading conditions and *r*/*x* ratios along with conclusion are presented.

2. Solution methodology

A single-line diagram of a radial distribution network (balanced and neglecting charging capacitances) is shown in Fig. 1(a). Table 1 shows the branch number, sending-end node and receiving-end node of Fig. 1(a). The directed graph of a radial network allows detection of the path of power flow from the reference node to the leaf end. The directed graph is represented by a *nodes-by-nodes* sparse matrix *S*, which is graphed as a bio-graph in Fig. 1(b) for the network of Fig. 1(a). Traversing the directed graph in depth-first search mode, the power flow paths (downstream nodes for each node including the node itself) can be detected [32,33]. A *BN* connection *branches by nodes* matrix is constructed, based on the discovered paths. The *PM* uses the two matrices *BIBC* and *BCBV*, to find out the solution satisfying the need of distribution automation [32]. The Appendix summaries the relationship between the bus current injections and branch current (*BIBC*) and the relationship between branch currents and voltage drops along branches (*BCBV*).

3. Proposed solution algorithm

To make proper use of graph theory number the branches, such that a branch between top *kk*th node and bottom $(kk+1)$th node of radial network, has the *k*th downstream node number – 1. This result in tabulation of the input information for both line data and node data in such way that the receiving end node will be in an ascending order. The vector of downstream nodes for each node in the original network graphs are obtained by graph traversing [32]. The number of downstream nodes for each node including the node itself is the length of this vector. Build the *BN* matrix from the vectors of downstream nodes. The rows of the bus-injection to branch-current (*BIBC*) are obtained through traversing, depth-first search the directed graph by following adjacent nodes starting from