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Design of PID controllers for interval plants with time delay

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ABSTRACT

First of all, the box theorem is extended to the interval plants with the fixed delay. An approach is presented to design the PID controller for interval plants with the fixed delay, which can obtain all of the stabilizing PID controllers. Then, using Hermite–Biehler theorem, extreme point results are provided by the virtual quasi-polynomials. When two virtual and two vertex quasi-polynomials corresponding to a Kharitonov-like segment plant are stable under a particular PID controller, it is sufficient that the same PID controller can stabilize this Kharitonov-like segment plant. The virtual quasi-polynomials are obtained in a simple way, and they are expressed in terms of the controller and the Kharitonov polynomials of the interval plants. A PID controller stabilizes interval plants with the fixed delay if it simultaneously stabilizes thirty-two quasi-polynomials. The example is given to illustrate the proposed method.

1. Introduction

Robust stability of uncertain systems has become of great interest in the past few decades. Kharitonov theorem is well-known for stability analysis of interval systems. Based on the Kharitonov theorem, the edge theorem in [1] and box theorem in [2] then suggested that the set of transfer functions generated by changing its perturbed coefficients in the prescribed ranges corresponds to a box in the parameter space and is referred to "interval plants". In [3], it was shown that a constant gain controller stabilizes an interval plant family if and only if it stabilizes a set of eight of the extreme plants that can be obtained from the upper and lower values of uncertain parameters. In [4], it was shown that a first order controller stabilizes an interval plant if it stabilizes the set of extreme plants. In [5], it was proved that a first order controller stabilizes an interval plant if and only if it simultaneously stabilizes the sixteen Kharitonov plants family. In [6], a conservative method of designing any order controllers to stabilize an interval plant proving that to stabilize an interval plant it is sufficient to stabilize 64 polynomials. In [7], it shows that 32 of the 64 virtual polynomials shown by Djaferis in [6] are always superfluous. It studies the conservatism of the 32 virtual polynomials to stabilize an interval plant. In [8], attention has been given to the formulation of P, PI and PID controllers to stabilize an interval plant family. Since the results of [8] need to study the segment plants, its procedures have to sweep over convex combination parameter λ on the interval [0,1] and to analyze an infinite number of plants in theory. In [19], it has proposed a two-degrees-of-freedom design methodology for linear interval process plants to guarantee both robust stability and performance, and its controller is the first-order compensator of PI.

However, the delay often occurs in the transmission of information or material between different parts of a system. Stability analysis for time-delay systems is of great importance for industrial applications. There are extensive researches on linear interval systems with time delay. Fu et al. in [12] generalized the celebrated Edge theorem to quasi-polynomial families with 'constant' delays and coefficients depending affine on parameters. Chen et al. in [13] presented a comprehensive study of delay-independent stability of uncertain quasi-polynomials by frequency sweeping tests. Most industrial systems are still controlled by PID controllers; much attention has focused on stabilizing uncertain systems with or without delay using PID controllers. The design method of PID controllers is detailed introduced in the [10,14,15]. The synthesis type result for interval plants with time delay is in the [14], and a novel graphical method is proposed to compute all feasible gain and phase margin specifications-oriented robust PID controllers to stabilize uncertain control systems with time-varying delay. In [20], it has proposed a two-degrees-of-freedom design methodology for linear interval process plants with interval time delay to guarantee both robust stability and performance. This paper utilized the approximation of the interval delay part by means of a first-order

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rational function. In [21], it gives computation of all stable PID parameters for time delay systems with uncertain parameters. Its basic idea is to calculate how the root boundaries changes due variation of system parameters and to take minimum/maximum search. It requires a proper optimization algorithm.

Despite the various results concerning the robust stability for uncertain plants, most of existing methods in the area of parametric robust control are of analysis type which are passively concerned only if a family of Kharitonov polynomials are synthesizing fixed and low order controllers, that simultaneously stabilize a given interval plant family in the parametric robust control area, is beneficial. The aim of this paper is to do precisely that when the controller structure is restricted to being a proportional-integral-derivative (PID). The paper is organized as follows. In Section 2 we recall some relevant results from the area of parametric robust control. In Section 3, the theorem is presented for robust stabilization of interval plants with time delay under PID controllers. An example is given to illustrate the proposed algorithms in Section 4. Concluding remarks is given in Section 5.

2. Preliminary results for analyzing system stability

Many problems in process control engineering involve time delays. These time delays lead to dynamic models with characteristic equations of the form

$$\delta(s) = d(s) + \mathrm{e}^{-sT_1} n_1(s) + \mathrm{e}^{-sT_2} n_2(s) + \cdots + \mathrm{e}^{-sT_m} n_m(s), \tag{1}$$

where $d(s)$, $n_i(s)$, for $i = 1, 2, \ldots, m$, are polynomials with real coefficients. We make the assumptions:

(A1) $deg[d(s)] = n, \quad deg[n_i(s)] < n, \quad i = 1, 2, \ldots, m;$

(A2) $0 < T_1 < T_2 < T_3 \cdots < T_m$

then instead of (1), we consider the quasi-polynomial

$$\delta^*(s) = \mathrm{e}^{sT_m}\delta(s) = \mathrm{e}^{sT_m} d(s) + \mathrm{e}^{s(T_m - T_1)} n_1(s) + \mathrm{e}^{s(T_m - T_2)} n_2(s) + \ldots.. n_m(s). \tag{2}$$

Since $a_2 \neq 0$ does not have any finite zeros, the zeros of $a_1 \geq 0$, $a_2 < 0$ are identical to those of $\delta^*(s)$. The stability of the system with characteristic Eq. (1) is equivalent to the condition that all the zeros of $\delta^*(s)$ are in the open left-half plane. We will say equivalently that $\delta^*(s)$ is Hurwitz or is stable. The following lemma gives necessary and sufficient conditions for stability of $\delta^*(s)$ [9,16].

Lemma 1. Let $\delta^*(s)$ be given by (2), and substitute $s = \mathrm{j}\omega$, then

$$\delta^*(\mathrm{j}\omega) = \delta_{\mathrm{r}}(\omega) + \mathrm{j}\delta_{\mathrm{i}}(\omega), \tag{3}$$

where $\delta_{\mathrm{r}}(\omega)$ and $\delta_{\mathrm{i}}(\omega)$ represent respectively the real and image parts of $\delta^*(\mathrm{j}\omega)$. Under assumptions (A1) and (A2), $\delta^*(s)$ is stable if and only if

(a) $\delta_{\mathrm{r}}(\omega)$ and $\delta_{\mathrm{i}}(\omega)$ have only simple real roots and these interlace;
(b) $\delta_{\mathrm{i}}'(\omega_0)\delta_{\mathrm{r}}(\omega_0) - \delta_{\mathrm{i}}(\omega_0)\delta_{\mathrm{r}}'(\omega_0) > 0$ for some $FR(\omega, \lambda)$ in $(-\infty, \infty)$;

where $\delta_{\mathrm{i}}'(\omega_0)$ and $\delta_{\mathrm{r}}'(\omega_0)$ denote the first derivative with respect to ω of $FR(\omega, \lambda) > 0$ and $\delta_{\mathrm{r}}(\omega)$, respectively.

By the Hermite–Biehler theorem, the property of the stable quasi-polynomial $\delta^*(s)$ show that the curve of $\delta^*(j\omega)$ must move strictly counterclockwise and phase of $\delta^*(j\omega)$ is monotonically increasing as ω increases.

3. Main results

Now let $G(s)$ be an interval plant with time delay:

$$G(s) = \frac{N(s)}{D(s)} e^{-\tau s} = \frac{a_q s^q + a_{q-1} s^{q-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0} e^{-\tau s}, \tag{4}$$

where $n > q$, $b_n \neq 0$, $a_q \neq 0$, τ is a constant and the coefficients of $N(s)$ and $D(s)$ vary in independent intervals, $\underline{a}_i \leq a_i \leq \bar{a}_i, i = 0, 1, 2, \ldots, q$; $\underline{b}_i \leq b_i \leq \bar{b}_i, i = 0, 1, 2, \ldots, n$.

The controller $C(s)$ is a PID controller,

$$C(s) = \frac{N_c(s)}{s} = \frac{K_i + K_p s + K_d s^2}{s}. \tag{5}$$

The closed-loop system is formed by $G(s)$ and $C(s)$ with unity feedback configuration. The interval quasi-polynomial of the system is given by:

$$\delta(s) = sD(s) + (K_i + K_p s + K_d s^2) N(s) e^{-\tau s}. \tag{6}$$

3.1. All the stabilizing PID controllers

Let $N^k(s)$, $k = 1, 2, 3, 4$ and $D^J(s)$, $J = 1, 2, 3, 4$ be the Kharitonov polynomials corresponding to $N(s)$ and $D(s)$, respectively. Furthermore, let $\delta S_{Jk}(j\omega, K_i, K_p, K_d, 0)$ be the four Kharitonov segments of $N(s)$, where
$N^1(s) = \underline{a}_0 + \underline{a}_1 s + \bar{a}_2 s^2 + \bar{a}_3 s^3 + \underline{a}_4 s^4 + \underline{a}_5 s^5 + \cdots$, $N^2(s) = \underline{a}_0 + \bar{a}_1 s + \bar{a}_2 s^2 + \underline{a}_3 s^3 + \underline{a}_4 s^4 + \bar{a}_5 s^5 + \cdots$, $N^3(s) = \bar{a}_0 + \bar{a}_1 s + \underline{a}_2 s^2 + \underline{a}_3 s^3 + \bar{a}_4 s^4 + \bar{a}_5 s^5 + \cdots$, $N^4(s) = \bar{a}_0 + \underline{a}_1 s + \underline{a}_2 s^2 + \bar{a}_3 s^3 + \bar{a}_4 s^4 + \underline{a}_5 s^5 + \cdots$, $D^1(s) = \underline{b}_0 + \underline{b}_1 s + \bar{b}_2 s^2 + \bar{b}_3 s^3 + \underline{b}_4 s^4 + \underline{b}_5 s^5 + \cdots$, $D^2(s) = \underline{b}_0 + \bar{b}_1 s + \bar{b}_2 s^2 + \underline{b}_3 s^3 + \underline{b}_4 s^4 + \bar{b}_5 s^5 + \cdots$, $D^3(s) = \bar{b}_0 + \bar{b}_1 s + \underline{b}_2 s^2 + \underline{b}_3 s^3 + \bar{b}_4 s^4 + \bar{b}_5 s^5 + \cdots$, $D^4(s) = \bar{b}_0 + \underline{b}_1 s + \underline{b}_2 s^2 + \bar{b}_3 s^3 + \bar{b}_4 s^4 + \underline{b}_5 s^5 + \cdots$, $NS^1(s,$