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A passivity-based approach to reset control systems stability[☆]

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ABSTRACT

The stability of reset control systems has been mainly studied for the feedback interconnection of reset compensators with linear time-invariant systems. This work gives a stability analysis of reset compensators in feedback interconnection with passive nonlinear systems. The results are based on the passivity approach to $\mathcal{L}_2$-stability for feedback systems with exogenous inputs, and the fact that a reset compensator will be passive if its base compensator is passive. Several examples of full and partial reset compensations are analyzed, and a detailed case study of an in-line pH control system is given.

1. Introduction

Reset control system design was initiated fifty years ago with the work of Clegg [1], who introduced a nonlinear integral feedback controller based on a reset action of the integrator, the so-called *Clegg integrator*. The reset action amounts to setting the integrator output equal to zero whenever its input is zero. In this way a faster system response without excessive overshoot may be expected, thus possibly overcoming a basic limitation of the standard linear integral feedback. This has spurred the development of several other nonlinear compensators, all based on describing function analysis. Furthermore, in a series of papers by Horowitz and co-workers [2,3] reset control systems have been advanced by introducing the first-order reset element (FORE).

One of the main drawbacks of reset compensators is that the stability of the feedback system is not always guaranteed by the stability of the underlying linear time-invariant (LTI) system without reset action. In fact it is well known, and easily illustrated, that the reset action can destabilize a stable LTI feedback system. Recently, the problem for linear reset control systems has been successfully addressed in [4,5] for general reset compensators, allowing full or partial state reset. As a result stability of the reset control system can be checked by the (strictly) positive realness of a certain transfer matrix H_β, referred to as the H_β-condition.

Reset control systems can be also regarded as a special case of *hybrid systems*, or as systems with impulsive motion. From this perspective, the recent work [6] addressed the stability problem of these types of systems with the goal of analyzing the stability of switching between LTI controllers. Furthermore, the H_β-condition has been relaxed in [7] to obtain a less restrictive Lyapunov stability condition. The papers [8,9] derive conditions based on the reset times that can be used both for stable and unstable linear systems.

Regarding the $\mathcal{L}_2$-stability of reset systems with inputs, a number of papers have appeared that give results for particular cases of reset compensators and/or inputs. The work [7] approaches the problem for compensators in which its output has the same sign as its input, and the zero reference case is considered. In addition, in [10] $\mathcal{L}_2$-stability conditions for the case of nonzero references are given. The conservatism given by H_β-condition is improved for these kinds of systems.

On the other hand, dissipative systems theory was developed in [11], where the concept of a *passive system*, originating from electric circuit theory and mechanical systems, was extended to abstract systems. A main theorem in this context is the fact that the feedback interconnection of two passive nonlinear systems is again a passive system. Passivity techniques have been shown to be a powerful tool for nonlinear control, see e.g. [12]. Dissipative systems theory has been developed for hybrid systems in [13], where notions such as *supply rate* have been extended to the hybrid case. We also refer to [14–17] for work on passivity of

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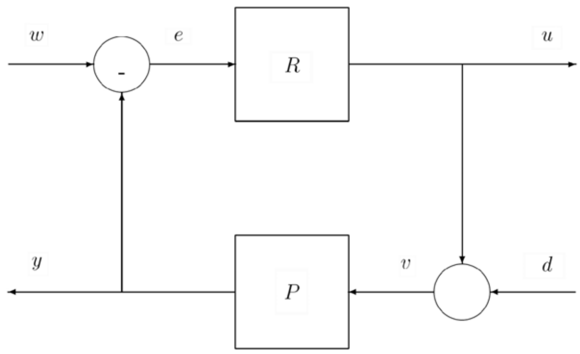

Fig. 1. Reset controller R applied to a LTI plant.

hybrid systems. In spite of the fact that a single-input single-output approach to passive systems theory will not be sufficient for more general hybrid systems, this paper will show how several passivity properties can be obtained for reset compensators. In [13], this kind of impulsive systems are referred to as *input-dependent impulsive dynamical systems*.

The goals of this work are: (a) to obtain stability conditions that are applicable to feedback interconnections of linear compensators with reset action and nonlinear plants; (b) to find passive reset compensators that can be used in passive control techniques. Passivity conditions for stability will be developed, which are easily checked on the linear compensator *without* reset action.

The structure of the work is the following. In Section 2, a description of the problem setup is given and some basic results about passivity theory are recalled. Section 3 gives the main results about the passivity properties of reset compensator, which are used to show $\mathcal{L}_2$-stability with respect to reference and perturbation inputs. In Section 4, an application to an industrial nonlinear plant is developed.

2. Preliminaries and problem setup

This work approaches the stability problem of reset control systems with inputs using general passivity theory. We consider the feedback system given by Fig. 1, where w and d are the reference and disturbance inputs, respectively. R is a single-input single-output (SISO) reset compensator to be defined later on, and P is a single-input single-output (SISO) plant. The set $\mathcal{L}_2$ consists of all measurable functions $f(\cdot):\mathbb{R}_+\to\mathbb{R}$ such that $\int_0^\infty |f(t)|^2 \mathrm{d}t<\infty$, being the $\mathcal{L}_2$-norm $\|\cdot\|:\mathcal{L}_2\to\mathbb{R}_+$ defined by $\|f\|=\left(\int_0^\infty |f(t)|^2\mathrm{d}t\right)^{\frac{1}{2}}$.

The feedback interconnection in system (Fig. 1) is given simply by

$$e(t)=w(t)-y(t),\qquad u(t)=v(t)+d(t). \tag{1}$$

The feedback system of Fig. 1 is called $\mathcal{L}_2$-stable (with respect to inputs w and d) if for every input signals $w\in\mathcal{L}_2$ and $d\in\mathcal{L}_2$ the outputs $u\in\mathcal{L}_2$ and $y\in\mathcal{L}_2$. In addition, it is finite-gain stable if there exists a positive constant $\gamma>0$ such that $\|y\|^2+\|u\|^2\le\gamma(\|w\|^2+\|d\|^2)$.

The plant P is represented by the state-space model

$$P:\begin{cases}\dot{x}_p=f(x_p,u), & u\in\mathbb{R}\\ y=g(x_p,u), & y\in\mathbb{R}\end{cases} \tag{2}$$

where n_p is the dimension of the state x_p, $f:\mathbb{R}^{n_p}\times\mathbb{R}\to\mathbb{R}^{n_p}$ is locally Lipschitz, $g:\mathbb{R}^{n_p}\times\mathbb{R}\to\mathbb{R}$ is continuous, $f(0,0)=0$, and $g(0,0)=0$. In addition, following the framework given in [13], the dynamics of the reset compensator is described by three elements: (a) a continuous-time dynamical equation, (b) a difference equation, and (c) a reset law. During time intervals in which the reset law is not applied, the system evolves in a continuous fashion; otherwise, when the resetting law is applied, the system undergoes a *jump*. We will throughout consider reset compensators R that consist of an LTI compensator (the so-called *base linear compensator*) together with a reset action, given by the following impulsive differential equation *(IDE)*:

$$R:\begin{cases}\dot{x}_r=A_r x_r+B_r e, & e\neq 0\\ x_r^+=A_\rho x_r, & e=0\\ u=C_r x_r+D_r e\end{cases} \tag{3}$$

where n_r is the dimension of the state x_r, A_ρ is a diagonal matrix with diagonal elements equal to zero for the state components to be reset, and equal to one for the rest of the compensator states, n_ρ is defined as the dimension of the reset subspace, and $n_{\overline{\rho}}$ is defined as the dimension of the non-reset subspace ($n_\rho+n_{\overline{\rho}}=n_r$). When $A_\rho=0$, R will be referred to as *full reset compensator*; otherwise, it will be referred to as *partial reset compensator*.

The first equation in (3) describes the continuous compensator dynamics at the non-reset time instants, while the second equation gives the reset operation as a jump of the compensator state at the reset instants. Note that reset time instants occur when the compensator input is zero. The base compensator is simply obtained by omitting the reset actions in (3), and thus has the transfer function $R_{bl}(s)=C_r(sI-A_r)^{-1}B_r+D_r$. Henceforth, whenever a transfer function is used to describe a reset compensator, this means that its base linear compensator has this transfer function. Furthermore, we will use the notations x_r^+ or $x_r(t^+)$ for the value $x_r(t+\tau)$ with $\tau\to 0^+$.

Impulsive systems such as (3) are a special case of hybrid systems, for which it is well that phenomena like Zeno behavior and beating may occur [13]. To avoid these phenomena, we will assume throughout this paper that the solutions to (3) are *time regularized* (see for example [7] and the references therein), which means that the reset law is switched off for a time interval of length $\Delta_m>0$ after each reset time. Thus formally speaking we consider the following reset system

$$R:\begin{cases}\dot{\Delta}=1, & \dot{x}_r=A_r x_r+B_r e, & e\neq 0 \text{ or } \Delta<\Delta_m\\ \Delta^+=0, & x_r^+=A_\rho x_r, & e=0 \text{ and } \Delta\ge\Delta_m\\ u=C_r x_r+D_r e\end{cases} \tag{4}$$

with zero initial conditions: $\Delta(0)=0$, $x_r(0)=0$. As a consequence of time regularization there will exist for any input e only a *finite* number of reset times on any finite time interval, hence excluding Zeno behavior. Furthermore, on the infinite time interval $[t_0,\infty)$ there will exists a countable set $\{t_1,t_2,\dots,t_k,\dots\}$ where $t_{k+1}-t_k\ge\Delta_m$ for all $k=1,2,\dots$, and in addition the constant Δ_m does not depend on the input e.

In contrast to the works [7] and [18], where the reset action is active when input and output have a different sign, the original definition of reset according to [1,3,2,5] has been used here. Note that the definition of Clegg integrator proposed in [18] is equivalent to the original in [1] in the case of zero initial condition. In addition, although the definition given in [18] has advantages in some particular cases, it cannot be applied to partial reset systems. This is the main reason why the original definition has been used in this work.

A system $H:\mathcal{L}_{2,e}\to\mathcal{L}_{2,e}$, with input u and output $y=Hu$ is said to be *passive* if there exists a constant $\beta\le 0$ such that

$$\int_0^T u^\top(t)y(t)\mathrm{d}t\ge\beta,\quad \forall T\ge 0,\ \forall u\in\mathcal{L}_2. \tag{5}$$

If there are constants $\delta\ge 0$ and $\epsilon\ge 0$ such that

$$\int_0^T u^\top(t)y(t)\mathrm{d}t\ge\beta+\delta\int_0^T u^\top(t)u(t)\mathrm{d}t+\epsilon\int_0^T y^\top(t)y(t)\mathrm{d}t, \tag{6}$$