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Influence of sampling on the tuning of PID controller parameters

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Abstract: The paper deals with an analysis of automatic control system with continuous and discrete PID controllers. A method of tuning the parameters of the continuous controller is presented, which is optimal according to the ITAE criterion. The behavior of control systems with discrete controllers whose parameters were tuned using the mentioned method are described. The impact of changes in the sampling period of controlled signal on the control quality is shown. Changes of the values of optimal parameters of discrete PID controllers in relation to changes of the sampling rate of controlled signal are characterized.

Keywords: PID controllers, tuning, optimal control, sampling rates.

1. INTRODUCTION

The general rules of the sampling period selection depend on the parameters of the control object model. The parameters are often: T_{max} – the dominant time constant, L – the transport delay time constant, T – inertia time constant (Kalman et al. 1958), (Astrom et al. 1984). There are also known rules determining sampling period with respect to the control quality indicators such as: t_s – settling time and t_r – rise time, (Isermann 1981), (Astrom et al. 1984). These rules allow one to estimate the signal sampling period with respect to the identified controller parameters (T_i – integration time, T_d – differentiation time constant) and are presented in: (Astrom et al. 1984), (Fertik 1975). These rules do not specify precisely what value of sampling period Δt ought to be used. They allow one to only roughly estimate the value of interval Δt.

It was assumed that the continuous control system is the reference system. It makes it easier to analyze the impact of sampling period of control signal on the control quality of the discrete system and the to choose of the optimal settings of discrete controllers. In the continuous system, the controller constantly monitors the controlled signal (process value) and the reference signal (setpoint value). On the basis of these signals it generates a control signal.

The settings of PI and PID controller are often selected using methods that are designed for continuous controllers (Ziegler et al. 1942), (Astrom et al. 1984), (Astrom et al. 1995). Badly selected continuous controller parameters can cause poor quality of control. The quality of control can deteriorate even more if the selected settings are used with a controller which responds to the input signals periodically – a discrete controller. To avoid this, the controller parameters are selected using an optimization method taking into account the sampling period. Such a method was proposed in (Wcislik et al. 2011) and it is briefly described in the next section.

2. OPTIMAL SETTINGS OF CONTINUOUS PID CONTROLER

The control system with negative feedback shown in figure 1 was analyzed as a basic control system.

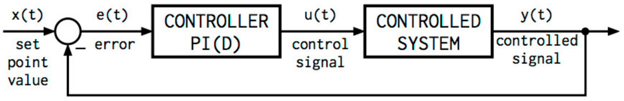

Fig. 1. The diagram of the basic automatic control system with continuous controller.

It was analyzed in (Wcislik et al. 2011). It was assumed that PID controller has the form:

$$G_{c(s)} = K_c\left[1 + 1/(sT_i) + (sT_d)/(s(T_d/N)+1)\right] \qquad (1)$$

where: K_c – proportional gain, T_i – integral time, T_d – derivative time, N – dimensionless coefficient.

The value of the dimensionless coefficient N is determined by bibliography analysis. Usually the value of the coefficient is in the range of 2 to 30 (O'Dwyer 2006). It was assumed that $N = 20$ (Wcislik et al. 2011).

The dynamics of the controlled system is approximated by a first-order inertial model with transport delay.

$$G_{(s)} = K\,e^{-sL}/(1+sT) \qquad (2)$$

where: K – static gain, T – inertia time constant, L – transport delay time constant.

The model description can map the dynamics of a wide range of industrial processes with satisfactory accuracy. It also makes it possible to model the steady state. The presence of a transport delay allows an approximation of potentially

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unstable processes. The ITAE was selected as an optimality criterion:

$$ITAE = \int_0^t t\left|e_{(t)}\right|dt \tag{3}$$

The procedure for the selection of the optimal settings of PI and PID controllers consists of a few steps. First, the proportional and derivative parts of the PID controller are disconnected. For the PI controller, only the proportional part is disconnected. Next, the gain of the integral part is increased to get the closed loop system to border stability. At this stage, the controller ultimate gain K_i and the sustained oscillation angular frequency ω_{osc} of the controlled variable y are assessed. On the basis of these parameters the time constant T of an approximating model is identified.

$$T = \sqrt{(K K_i)^2 - \omega_{osc}^2} / \omega_{osc} \tag{4}$$

Then the coefficient $\theta = L/T$ is calculated

$$\theta = arcctg(\omega_{osc} T) / \omega_{osc} T \tag{5}$$

The optimal settings of the PI controller were defined in (Wcislik et al. 2011):

$$\begin{aligned} K_c &= \left[10^{\left(0.49/\sqrt{\theta}\right)-0.67}\right]/K \\ T_i &= T \cdot \left[0.0058\theta^2 + 0.31\theta + 0.91\right] \end{aligned} \tag{6}$$

and of the PID controller:

$$\begin{aligned} K_c &= \left[10^{\left(0.81/\sqrt[3]{\theta}\right)-0.79}\right]/K \\ T_i &= T \cdot \left[0.4\theta + 0.97\right] \\ T_d &= T \cdot \left[0.48\sqrt{\theta} - 0.16\right] \end{aligned} \tag{7}$$

The equations (6) and (7) were obtained using the approximation of the set of PI and PID optimal settings. The least squares method was used in this purpose. The obtained formulas provide acceptable accuracy for $\theta = <0.2, 2>$.

Examples of step responses of the continuous systems with PID controllers were shown in figure 2. The controllers parameters were selected using the Ziegler-Nichols method (Ziegler et al. 1942) and the proposed method (Wcislik et al. 2011). The controlled system has a transfer function described by (2) with $\theta = L/T = 0.2$.

The use of the proposed method causes a slight increase of the rise time as well as a decrease of the settling time and the overshoot value. The control quality is significantly better than for the Ziegler-Nichols method.

The transients for the proposed method in figure 2 have some pulse disturbances. They arise from an interaction between the derivative part of the PID controller and the transport delay of the controlled system.

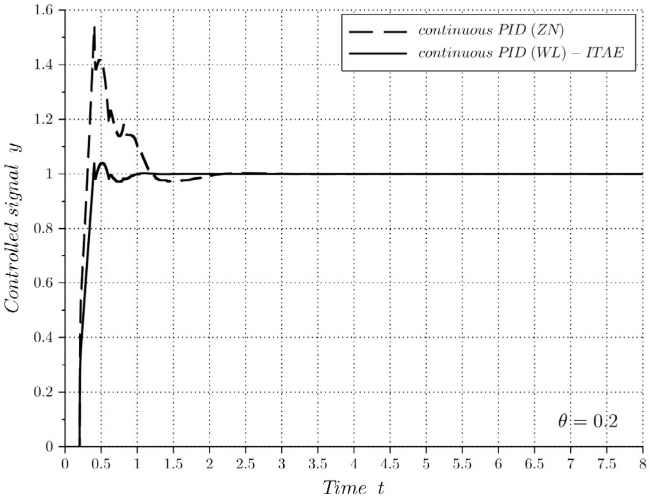

Fig. 2. The transients of the continuous control system with the PID controller.

3. OPTIMAL SETTINGS OF DISCRETE TIME PID CONTROLER

The block diagram of an automatic control system with discretized control signal with sapling period Δt is shown in figure 3.

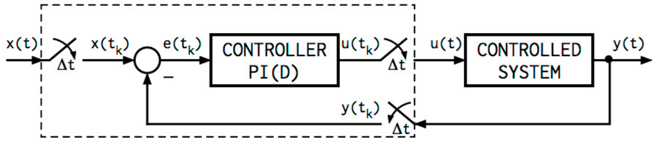

Fig. 3. Discretized automatic control system.

Simulation of the system presented in figure 3 requires the separation of the elements that are solved with different periods. The summation node as well as the controller are solved with period Δt. For the continuous part of the system, the model of the controlled system is solved with the step δt. The step δt may be either fixed or variable. It depends on the chosen method of solving differential equations describing the controlled system.

In SIMULINK environment, the above discrete control system requires two ZOH (zero-order hold) extrapolators, which must be placed before and after the continuous model of the controller (fig. 4).

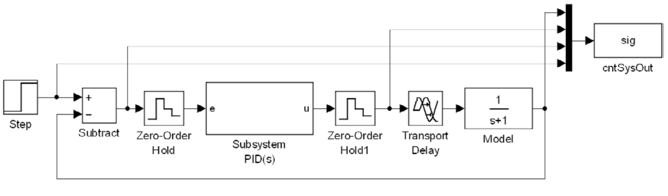

Fig. 4. Diagram of the discrete control system in SIMULINK.

Simulation results show that the insertion of the ZOH extrapolators into the continuous system (fig. 4) causes system instability, even if the original continuous system was stable and the transient of the controlled signal was optimal.