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Robust PID Controller Design for Performance Based on Ultimate Plant Parameters

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Abstract: The paper deals with a new robust PID controller design method based on integrating requirements on transient performance into the popular frequency-domain Ziegler-Nichols design approach. The developed method provides support to the designer by converting identified ultimate plant parameters into PID controller parameters using variable weights that depend on expected maximum overshoot η_{max} and settling time t_s of the closed-loop step response. An extension of this method is proposed which enables to design PID controller guaranteeing robust stability and nominal performance for the uncertain plant modeled using unstructured uncertainty. Fulfillment of performance specification is guaranteed for the nominal model as well as for the worst-case one. The main advantage of the proposed method is a direct integration of performance requirements in the design procedure. Effectiveness of the proposed robust PID design method is verified by simulations and experiments on the real plant.

Keywords: performance evaluation, overshoot, settling time, ultimate frequency, phase margin

1. INTRODUCTION

Quick computation of controller parameters and simple algorithmisation are main attributes due to which the frequency response Ziegler-Nichols method (Ziegler and Nichols, 1942) is widely used for tuning PID controllers implemented in industrial control loops. However, it is a closed design method not allowing the designer to modify the performance with respect to the specific technological process (Bucz et al., 2014).

There are several similar methods extending the Ziegler-Nichols frequency method (McAvoy and Johnson, 1967), (Atkinson and Davey, 1968), (Tinham, 1989). According to the original Ziegler-Nichols frequency method the PID tuning rules are of the form $\Theta_{PID}=(P,T_i,T_d)=(\alpha_1 K_c,\alpha_2 K_c,\alpha_3 T_c)= =(0.6K_c,0.5K_c,0.125T_c)$, where $(\alpha_1,\alpha_2,\alpha_3)$ are weights of ultimate plant parameters T_c and K_c, (Pettit and Carr, 1987) propose three settings $(\alpha_1,\alpha_2,\alpha_3)=(1,0.5,0.125)$, $(0.5,1,0.167)$, $(0.67,1,0.167)$. The first two leading to underdamped and aperiodic responses of the output variable, respectively, and the third one to a response on the aperiodicity border. Rather than fixed values (Karaboga and Kalinli, 1996) propose intervals $\alpha_1\in\langle 0.32,0.6\rangle$, $\alpha_2\in\langle 0.213,1.406\rangle$, $\alpha_3\in\langle 0.133,0.469\rangle$, however, without any recommendations with respect to expected performance. However, assessment of expected performance achieved by PID controllers tuned according to these methods is very approximate and only representative (Åström and Hägglund, 2000).

To remove this drawback, the proposed modified frequency response Ziegler-Nichols method allows to achieve specified maximum overshoot $\eta_{max}\in\langle 0\%,50\%\rangle$ and settling time $t_s\in\langle 7/\omega_c,22/\omega_c\rangle$ of the closed-loop response to the setpoint step change, where ω_c is the plant critical frequency. Fulfillment of performance specification is guaranteed for the nominal model as well as for the worst-case one.

The paper is organized as follows: Section 2 describes the classical frequency response Ziegler-Nichols method and demonstrates its modification with respect to transient performance requirements. Achieved performance is assessed and modified Ziegler-Nichols tuning rules for various values of maximum overshoot and settling time are provided. In Section 3, an exact extension of the new method is proposed yielding robust PID design rules for uncertain systems modelled using unstructured uncertainty. The proposed method was verified via simulations on benchmark examples and on a real plant - a DC motor with variable load torque; the results are summarized in Section 4.

2. PID CONTROLLER DESIGN

2.1 Ziegler-Nichols frequency response method: principle and analysis

The frequency domain Ziegler-Nichols method (1942) is a direct engineering method with fast rejection of the disturbance $z(t)$ being most frequently cited in technical literature. To design a controller, only two characteristic parameters of the unknown plant are to be identified.

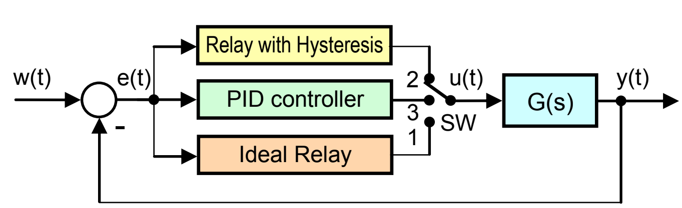

Fig. 1. Feedback control loop.

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Consider the feedback loop in Fig. 1; put the PID controller in proportional mode *(SW=3)* and increase the gain K of the controller $G_R(s)=K$ until the output $y(t)$ exhibits persistent oscillations; from them, the critical period T_c and the related critical gain K_c are read. If considering the standard interacting form of the PID controller

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{1}$$

where K is proportional gain, T_i, T_d are integral and derivative constants, respectively, coefficients of P, PI and PID controllers are calculated according to the Table 1.

Table 1. PID tuning rules according to the Ziegler-Nichols frequency response method

Controller	*K*	T_i	T_d	T_p
P	$0{,}5K_c$	-	-	T_c
PI	$0{,}45K_c$	$0{,}8T_c$	-	$1{,}4T_c$
PID	$0{,}6K_c$	$0{,}5T_c$	$0{,}125T_c$	$0{,}85T_c$

Relations in the last column of Table 1 can be used to estimate the dominant closed-loop dynamics T_p (Åström and Hägglund, 1995). According to the Ziegler-Nichols frequency response method, if the open-loop transfer function with the proportional controller (Fig. 1)

$$L(j\omega) = G(j\omega)G_R(j\omega) = KG(j\omega) \tag{2}$$

is at the limit of instability, it can be expressed in polar form according to the Nyquist condition

$$L(j\omega_c) = -1 = 1e^{-j180°}, \tag{3}$$

where $\omega_c=2\pi/T_c$ is the critical frequency of the plant. From comparison of (2) and (3) at $\omega=\omega_c$ and for $K=K_c$ results the complex equation

$$G(j\omega_c) = [1/K_c]e^{-j180°}, \tag{4}$$

which expresses position of the plant critical point $C=G(j\omega_c)$ with coordinates $\{\omega_c, 1/K_c, -\pi\}$ on the negative half-axis of the complex plane. This point is crossed by the frequency characteristics of the unknown plant. If we substitute the Ziegler-Nichols PID controller tuning rules from Table 1 into the frequency response transfer function of the PID controller

$$G_R(j\omega) = K\left[1 + j\left(T_d\omega - \frac{1}{T_i\omega}\right)\right] \tag{5}$$

and consider critical frequency, we obtain the complex number

$$G_R(j\omega_c) = 0{,}6K_c\left[1 + j\left(\frac{T_c}{8}\frac{2\pi}{T_c} - \frac{T_c}{0{,}5.T_c.2\pi}\right)\right] = 0{,}66K_c e^{j25{,}01°}$$

with a magnitude depending solely on the critical gain of the plant, and a constant argument. Hence, the PID controller designed by the Ziegler-Nichols method moves the critical point C of the plant with coordinates (4) into the fixed position in the complex plane $L(j\omega_c)=G(j\omega_c)G_R(j\omega_c)$

$$L(j\omega_c) = \left[\frac{1}{K_c}e^{-j180°}\right]\left[0{,}66K_c e^{j25°}\right] = 0{,}46e^{-j155°} = -0{,}6 - j0{,}28$$

which will be one point of the open-loop Nyquist plot L(jω) under the designed PID controller (see Fig. 2a).

2.2 Principle of the modified frequency response Ziegler-Nichols method for specified performance

The presented modified version of the Ziegler-Nichols method integrates performance requirements into its classical version (Bucz et al., 2014). The PID controller is tuned using the derived modification of the Ziegler-Nichols table which includes separate rules for adjusting controller coefficients for:

- maximum overshoot $\eta_{max} \in \{0\%, 10\%, 20\%, 30\%, 40\%, 50\%\}$,
- settling time $t_s \in \{7/\omega_c, 10/\omega_c, 13/\omega_c, 16/\omega_c, 19/\omega_c, 22/\omega_c\}$.

Principle of the proposed modification consists in moving the identified critical point of the plant $C=G(j\omega_c)=[-1/K_c j0]$ using PID controller into the complex plane point $L(j\omega_c)=x+jy$ which will be a point of the Nyquist plot $L(j\omega)$ of the designed open-loop (see Fig. 2b). This compensation is carried out at critical frequency ω_c of the plant. Coordinates x and y specifying the future position of the critical point C at ω_c will depend on the expected performance specified by the designer in terms of η_{max} and t_s. Mathematically, this compensation can be described by the open-loop transfer function at ω_c:

$$L(j\omega_c) = G(j\omega_c)G_R(j\omega_c) = x + jy\,. \tag{6}$$

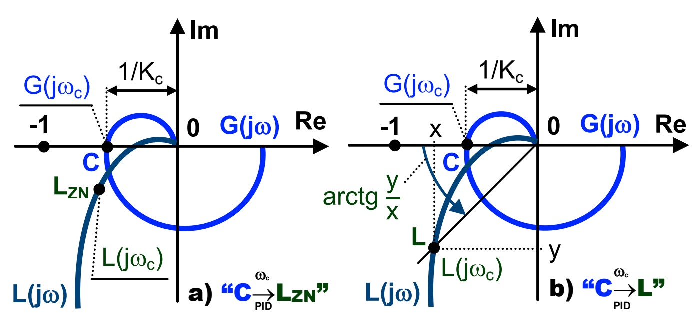

Fig. 2. Illustration of moving the critical point C into a) $L_{ZN}=[-0{,}6\text{-}j0{,}28]$ (by Z-N method); b) $L=[x+jy]$ (by the proposed method).

After substituting coordinates of the critical point C into (6), the controller transfer function $G_R(j\omega_c)$ turns into a complex number

$$G_R(j\omega_c) = \frac{L(j\omega_c)}{G(j\omega_c)} = -K_c(x + jy). \tag{7}$$

If equating (7) and the PID controller frequency transfer function (5), controller coefficients can be obtained from the complex equation at $\omega=\omega_c$

$$-K_c(x + jy) = K\left(1 + j\left(T_d\omega_c - \frac{1}{T_i\omega_c}\right)\right). \tag{8}$$

To calculate PID controller coefficients, following relations resulting from (8) are used