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Gain-scheduled PID controller design

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ABSTRACT

Gain scheduling (GS) is one of the most popular approaches to nonlinear control design and it is known that GS controllers have a better performance than robust ones. Following the terminology of control engineering, linear parameter-varying (LPV) systems are time-varying plants whose state space matrices are fixed functions of some vector of varying parameters. Our approach is based on considering that the LPV system, scheduling parameters and their derivatives with respect to time lie in a priori given hyper rectangles. To guarantee the performance we use the notion of guaranteed costs. The class of control structure includes centralized, decentralized fixed order output feedbacks like PID controller. Numerical examples illustrate the effectiveness of the proposed approach.

1. Introduction

Linear parameter-varying systems are time-varying plants whose state space matrices are fixed functions of some vector of varying parameters $\theta(t)$. Linear parameter varying (LPV) systems have the following interpretations:

– they can be viewed as linear time invariant (LTI) plants subject to time-varying known parameters $\theta(t) \in \langle \underline{\theta} \;\; \overline{\theta} \rangle$,
– they can be models of linear time-varying plants,
– they can be LTI plant models resulting from linearization of the nonlinear plants along trajectories of the parameter $\theta(t) \in \langle \underline{\theta} \;\; \overline{\theta} \rangle$ which can be measured.

For the first and third class of systems, parameter θ can be exploited for the control strategy to increase the performance of closed - loop systems. Hence, in this paper the following LPV system will be used:

$$\begin{aligned} \dot{x} &= A(\theta(t))x + B(\theta(t))u \\ y &= Cx \end{aligned} \tag{1}$$

where for the affine case

$$A(\theta(t)) = A_0 + A_1\theta_1(t) + \cdots + A_p\theta_p(t) \tag{2}$$

$$B(\theta(t)) = B_0 + B_1\theta_1(t) + \cdots + B_p\theta_p(t) \tag{3}$$

and $x \in R^n$ is the state, $u \in R^m$ is a control input, $y = R^l$ is the measurement output vector, A_0,B_0,A_i,B_i, $i = 1, 2, \ldots, p$, C are constant matrices of appropriate dimension, $\theta(t) \in \langle \underline{\theta} \;\; \overline{\theta} \rangle \in \Omega$ and $\dot{\theta}(t) \in \langle \underline{\dot{\theta}} \;\; \overline{\dot{\theta}} \rangle \in \Omega_t$ are vectors of time-varying plant parameters which belong to the known boundaries.

In the case of nonlinear dynamics a widely used idea among control engineers is to linearize the plant around several operating points and to use linear control tools to design a controller for each of these points. The actual controller is implemented using the gain scheduling approach. Success of such an approach depends on establishing the relationship between a nonlinear system and a family of linear ones. There are two main problems:

1 Stability results: stability of the closed-loop nonlinear system and of the closed-loop family of linear systems, when scheduled parameters are changes.
2 Approximation results which provide a direct relationship between the solution of closed-loop nonlinear systems and the solution of associated linear systems [1,2]

The main motivation for our work lies in [3–8], where in [3] the LPV controller is designed using the bounded real lemma for continuous and discrete time LPV systems such as to guarantee H_∞ performance.

Paper [4] discusses extensions of H_∞ synthesis techniques to allow for controller dependence on time-varying but measured parameters. In this case a higher performance can be achieved by control laws that incorporate measurements of θ to the control algorithm. Main results can be formulated as follows: Find a control structure such that the LPV controller satisfies closed-loop

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stability and minimizes of the induced L_2 norm of corresponding closed-loop systems. The author's approach [5] uses a bounding technique based on parameter-dependent Lyapunov function for design of PD controllers. Note that if LPV synthesis problem is solvable, then the induced L_2-norm of the closed-loop system is less than some given constant. The proposed approach represents generalization of the standard sub-optimal H_∞ control problem. In paper [6] the author shows that the performance of LPV systems with LPV controller can be improved by combining this LMI method with MPC techniques and optimizing the H_2 (H_∞) norm. The author [8] tackles the design problem of gain scheduled controllers for LPV systems via parameter-dependent Lyapunov function. The author proposed a new design method as a set LMIs with single line search parameters. The author tackles two problems: H_∞ type problem and H_2. Recently, [9] proposed the design method for the gain scheduled problem using a similar technique to [8]. In the above paper the LPV controller is given in time domain with the same or lower order than the LPV systems using H_∞ optimization approach. The gain scheduling controller design for discrete-time systems is given in [10]. Paper [11] presents the design of gain-scheduled PI controller, when the uncertainty of the system is assumed to be the difference between the nonlinear model and the nominal linear model. PI controller is designed using quadratic Lyapunov H_∞ performance where index γ is H_∞ norm of closed-loop system, considered as closed-loop performance measure. Minimizing γ via LMI the gain scheduled controller is obtained. In [12] the authors design a novel gain scheduling controller for synchronous generator. Improved stability analysis and gain scheduled controller synthesis for parameter-dependent systems are proposed in [7]. Sufficient conditions for robust stability as well as conditions for the existence of a gain-scheduled controller are given in terms of a set of LMIs. The author's approach is based on the notion of quadratic stability and linear fractional representation for parameter dependent systems. The survey of scheduled controller analysis and synthesis can be found in excellent papers [1] and [2].

In this paper our approach is based on:

- A consideration of the LPV systems (1). The scheduling parameters θ_i, $i = 1, 2, \ldots p$ and their derivatives with respect to time are supposed to lie in a priori given hyper rectangles.
- Affine quadratic stability (AQS) introduced by [13].
- To guarantee the performance we use the notion of guaranteed cost to optimize the given cost function.
- The class of control structure includes centralized, decentralized fixed order output feedback like PID controller.

The gain-scheduled controller design procedure is in the form of BMI. A feasible solution for closed-loop system ensures the affine quadratic stability [13] and guaranteed cost when the performance is defined in Q, R, S structure (see Eq. (10)).

Quadratic stability (one Lyapunov function with one constant positive definite matrix cover all affine controller design procedure) is more conservative than AQS in general. AQS (Lyapunov function has an affine structure like (2)) incorporates information about the rate of variation $\dot{\theta}(t)$ to reduce conservatism. As we mentioned, in this paper the AQS approach will be used.

Our notations are standard. $D \in R^{m\times n}$ denotes the set of real $m \times n$ matrices. I_m is an $m \times m$ identity matrix. If the size can be determined from the context, we will omit the subscript. $P > 0$ ($P \geq 0$) is a real symmetric, positive definite (semi-definite) matrix.

The paper is organized as follows. Section 2 brings preliminaries and problem formulation. The main result is presented in Section 3. In Section 4, numerical examples illustrate the effectiveness of the proposed approach.

2. Preliminaries and problem formulation

Suppose that the state-space representation of an LPV system with p independent scheduling parameters is governed by (1). The scheduling parameters θ_i and their derivatives with respect to time $\dot{\theta}_i$ are supposed to lie in given hyper rectangles Ω and Ω_t, respectively. For design of the I part of the controller system, Eq. (1) has to be augmented, see [15] and example 1. Without change of notation the new augmented matrices dimensions are $A(\theta) \in R^{(n+l)\times(n+l)}$, $B(\theta) \in R^{(n+l)\times m}$, $C \in R^{2l\times 2l}$ and $C_d \in R^{l\times l}$ is the output matrix for D part of controller. The output feedback gain-scheduled control law is considered for PID controller in the form

$$u(t) = F(\theta)y + F_d(\theta)\dot{y}_d = F(\theta)Cx + F_d C_d \dot{x} \tag{4}$$

where $y_d = C_d x$ is the output feedback for the D part of the controller,

$$F(\theta) = F_0 + \sum_{i=1}^{p} F_i\theta_i \quad \in R^{m\times 2l} \tag{5}$$

is the static output feedback gain scheduled matrix for the PI controller and

$$F_d(\theta) = F_{d_0} + \sum_{i=1}^{p} F_{d_i}\theta_i \quad \in R^{m\times m} \tag{6}$$

is the static output feedback gain scheduled matrix for the D part of controller.

Remark 1. Since the reference signal does not influence the closed-loop stability, we assume that it is equal to zero.

Remark 2. If the derivative part of the controller includes some filter, the model of this filter can be included in the system model.

The closed-loop system is then

$$[I - B(\theta)F_d(\theta)C_d]\dot{x} = [A(\theta) + B(\theta)F(\theta)C]x \tag{7}$$

$$A_d(\theta)\dot{x} = A_c(\theta)x \tag{8}$$

$$\dot{x} = A_{cd}(\theta)x \tag{9}$$

where

$$\begin{aligned} A_{cd}(\theta) &= A_d(\theta)^{-1}A_c(\theta)x \\ A_d(\theta) &= I - B(\theta)F_d(\theta)C_d \\ A_c(\theta) &= A(\theta) + B(\theta)F(\theta)C \end{aligned}$$

It is well known that the fixed order dynamic output feedback control design problem is a special case of the static output feedback problem. To access the performance quality a quadratic cost function [15] known from LQ theory is often used in the form

$$J = \int_0^\infty (x^T Qx + u^T Ru + \dot{x}^T S\dot{x})dt \tag{10}$$

with $Q = Q^T \geq 0$, $R > 0$ and $S = S^T \geq 0$. The guaranteed cost is defined in a standard way.

Definition 1. Consider system (1) with control algorithm (4). If there exists a control law u^* and a positive scalar J^* such that the closed-loop system (7) is stable and the value of closed-loop cost function (10) satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost and u^* is said to be guaranteed cost control law for system (1). □