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PID controller tuning for integrating processes

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ABSTRACT

Minimizing the integral squared error (ISE) criterion to get the optimal controller parameters results in a PD controller for integrating processes. The PD controller gives good servo response but fails to reject the load disturbances. In this paper, it is shown that satisfactory closed loop performances for a class of integrating processes are obtained if the ISE criterion is minimized with the constraint that the slope of the Nyquist curve has a specified value at the gain crossover frequency. Guidelines are provided for selecting the gain crossover frequency and the slope of the Nyquist curve. The proposed method is compared with some of the existing methods to control integrating plant transfer functions and in the examples taken it always gave better results for the load disturbance rejection whilst maintaining satisfactory setpoint response. For ease of use, analytical expressions correlating the controller parameters to plant model parameters are also given.

1. Introduction

The ability of proportional-integral-derivative (PID) controllers to meet most of the control objectives has led to their widespread acceptance in the control industry. A comprehensive summary of the PID tuning methods reported in the literature is given in [1]. It can be concluded from [1] that the number of tuning rules are much more for stable overdamped processes as compared to integrating processes. Several PI/PID tuning methods for integrating processes have been proposed in the literature [2–9]. Chien and Fruehauf [2] have proposed an internal model control (IMC) approach to select tuning constants of a PI controller for a process transfer function consisting of a pure integrator and a dead time. Tyreus and Luyben [3] have shown that the IMC based PI controller can lead to poor control performance if the closed loop time constant is not chosen properly and have proposed simple tuning rules based on the classical frequency response method to achieve maximum closed loop log modulus of 2 dB. Kookos et al. [5] have obtained the parameters of the PI controller by gain-phase margin method (KPGM) and by minimizing weighted ISE. Tuning rules for PI controller based on the maximum peak resonance specification is proposed in [6]. However, the above methods do not give settings for PID controllers.

The method proposed in [3] is extended for designing PID controllers in [4]. The desired control signal trajectory is used as a performance specification to design the PID controller in [8]. Chidambaram and Sree [9] have obtained the parameters of a PI, PID and PD controller by matching the coefficients of corresponding powers of s in the numerator and that in the denominator of the closed loop transfer function for a servo problem. The method proposed in [9] is further improved by Sree and Chidambaram [10] to avoid the oscillations in the system output for perturbation in the plant delay. Skogestad [11] has used the IMC framework to derive PI/PID tuning rules (SIMC) for various class of integrating processes. However, none of the above reported works give the settings for PID controller for integrating plus time delay (IPTD), integrating plus first order plus time delay (IFOPTD) and double integrating plus time delay (DIPTD) process models, respectively.

One of the methods of obtaining the controller parameters is by minimizing an integral criterion. The integral of squared error (ISE) has often been used for control system design since the integral can be evaluated analytically in the frequency domain. Although the ISE method provides a good way to obtain optimal PID controller settings, it weights all errors equally independent of time and hence results in a response with a relatively small overshoot but a long settling time. For integrating processes, the ISE criterion results in a PD controller which gives good servo response but fails to reject the load disturbances. Recently, Karimi et al. [12] have used the Bode's integrals to approximate the derivatives of amplitude and phase of a system with respect to frequency at a given frequency and these derivatives are then used to adjust the slope of the Nyquist curve at the gain crossover frequency to obtain the parameters of a PID controller using the modified Ziegler–Nichols method. However, the selection of gain crossover frequency and the slope of the Nyquist curve has not been discussed in [12]. In this work, it is shown that minimizing the ISE criterion so that the slope

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of the Nyquist curve attains a specified value at the gain crossover frequency gives satisfactory closed loop performance for a class of integrating processes.

The paper is organized as follows. Section 2 presents the problem formulation: process model, PID structure and the optimization problem. The expressions for the PID parameters in terms of plant model parameters are presented in Section 3. Simulation results are discussed in Section 4 followed by conclusions in Section 5.

2. Problem formulation

In this section, the controller equation is presented as well as the assumed process model structure and the optimization problem that is posed in order to tune the PID controller.

2.1. PID controller

The general parallel form of a PID controller is given by

$$G_c(s) = \frac{U(s)}{E(s)} = K_c\left(1 + \frac{T_d s}{T_f s + 1} + \frac{1}{T_i s}\right) \tag{1}$$

where K_c, T_i and T_d are the controller parameters to be obtained for the satisfactory closed loop performance of the system. $T_f = \alpha T_d$ and usually, a small value of the derivative filter constant (α) is considered in the literature. In this work α is set at 0.10. Further, $e(t)$ and $u(t)$ are the input and output signals of the PID controller, respectively.

2.2. Process model

For controller design purposes, the commonly encountered integrating processes are adequately described by low order transfer function models such as integrating plus time delay (IPTD),

$$G_p(s) = \frac{K_p e^{-\theta s}}{s} \tag{2}$$

integrating plus first order plus time delay (IFOPTD) and

$$G_p(s) = \frac{K_p e^{-\theta s}}{s(Ts+1)} \tag{3}$$

double integrating plus time delay (DIPTD)

$$G_p(s) = \frac{K_p e^{-\theta s}}{s^2}. \tag{4}$$

2.3. Selection of gain crossover frequency

The gain crossover frequency is related to the rise time and thus to the bandwidth of the closed loop system and hence, can be used as a measure of system performance [13]. Also, the slope of the Nyquist curve at ω_g gives a measure of the minimum distance of the Nyquist plot from the critical point (−1, 0) which indicates the system robustness. Hence, we can say that the slope of the Nyquist curve at gain crossover frequency drastically affects the performance and robustness of the closed loop system. In this subsection, an upper bound on the gain crossover frequency (ω_g) is obtained for the above mentioned class of integrating processes using the Åström's inequality proposed in [14].

The best achievable control performance for a process is limited by its non-minimum phase nature and to figure out this achievable performance, the process model is factorized into

$$G_p(s) = G_{mp}(s) G_{nmp}(s) \tag{5}$$

where $G_{mp}(s)$ is the minimum phase part and $G_{nmp}(s)$ the non-minimum phase part, with $|G_{nmp}(j\omega)| = 1$. Recently, Åström [14] has shown that the gain crossover frequency satisfies the following inequality

$$\arg G_{nmp}(j\omega_g) \geq -\pi + \phi_m - n_g \frac{\pi}{2} \tag{6}$$

where ϕ_m is the required phase margin and n_g the slope of the open loop gain at the crossover frequency. The open loop gain should have a slope of about −1 around the crossover frequency, with preferably steeper slopes before and after the crossover and hence n_g is assumed as −1 [15]. For the considered integrating process models, Eq. (6) reduces to

$$\theta\omega_g \leq \frac{\pi}{2} - \phi_m. \tag{7}$$

Typical values of ϕ_m range from 30° to 60° [16]. Therefore, in this work three values of ϕ_m, i.e. 30°, 45° and 60° are considered and the corresponding values of $\theta\omega_g$ (considering the equality sign in (7)) are 1.05, 0.78 and 0.52, respectively.

2.4. Loop slope adjustment

In this subsection, a relationship between T_i and T_d is obtained to achieve a user specified slope at any given frequency. The slope of the Nyquist curve of the loop transfer function $L(s) = G_p(s)G_c(s)$ at any frequency ω_o is equal to the phase of the derivative of $L(j\omega)$ at ω_o.

$$\frac{dL(j\omega)}{d\omega} = G_p(j\omega)\frac{dG_c(j\omega)}{d\omega} + G_c(j\omega)\frac{dG_p(j\omega)}{d\omega}. \tag{8}$$

Eq. (8) gives the derivative of the loop transfer function with respect to ω. Also, we have

$$\ln G_p(j\omega) = \ln\left|G_p(j\omega)\right| + j\angle G_p(j\omega). \tag{9}$$

Differentiating the above equation, we get

$$\frac{dG_p(j\omega)}{d\omega} = G_p(j\omega)\left\{\frac{d\ln\left|G_p(j\omega)\right|}{d\omega} + j\frac{d\angle G_p(j\omega)}{d\omega}\right\}. \tag{10}$$

The derivative of the controller (neglecting T_f) with respect to ω is given by

$$\frac{dG_c(j\omega)}{d\omega} = jK_c\left(T_d + \frac{1}{\omega^2 T_i}\right). \tag{11}$$

Substituting (1), (10) and (11) in (8), we get

$$\frac{dL(j\omega)}{d\omega} = G_p(j\omega)K_c\left\{j\left(T_d + \frac{1}{\omega^2 T_i}\right) + \left(1 + j\left(T_d\omega - \frac{1}{\omega T_i}\right)\right) \times \left(\frac{d\ln\left|G_p(j\omega)\right|}{d\omega} + j\frac{d\angle G_p(j\omega)}{d\omega}\right)\right\}. \tag{12}$$

The slope of the Nyquist curve at ω_o denoted by ψ is given by

$$\psi = \varphi_o + \arctan\left(\frac{1 + T_i T_d \omega_o^2 + s_p(\omega_o)\omega_o T_i + (T_i T_d \omega_o^2 - 1)s_a(\omega_o)}{s_a(\omega_o)\omega_o T_i - (T_i T_d \omega_o^2 - 1)s_p(\omega_o)}\right) \tag{13}$$

where

$$\varphi_o = \angle G_p(j\omega_o)$$

$$s_a(\omega_o) = \omega_o \left.\frac{d\ln\left|G_p(j\omega)\right|}{d\omega}\right|_{\omega_o}$$

$$s_p(\omega_o) = \omega_o \left.\frac{d\angle G_p(j\omega)}{d\omega}\right|_{\omega_o}. \tag{14}$$

Rearranging (13), we get the equation given in Box I.