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Robust Stabilization of Oblique Wing Aircraft Model Using PID Controller

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Abstract: The paper is focused on computation of all possible robustly stabilizing Proportional-Integral-Derivative (PID) controllers for oblique wing aircraft modelled as interval system. The main idea of the proposed method is based on Tan's technique for calculation of (nominally) stabilizing PI and PID controllers or robustly stabilizing PI controllers by means of plotting the stability boundary locus in either P-I plane or P-I-D space. Refinement of the existing method by consideration of 16 segment plants instead of 16 Kharitonov plants provides an elegant and efficient tool for finding all robustly stabilizing PID controllers for an interval system.

Keywords: Robust Stabilization, Oblique Wing Aircraft, PID Control, Interval Systems.

1. INTRODUCTION

The Proportional-Integral-Derivative (PID) control algorithms and their simplifications (P, I, PD and especially PI) comprise the great majority of contemporary industrial control applications. It has been reported that they represent over 95% of all practically applied controllers (Åström and Hägglund, 1995), (O'Dwyer, 2003). Thus, despite the existence of many more sophisticated control design methods and modern approaches, the effective tuning of PI and PID controllers is still very topical because it can bring significant saving on energy as well as expenses. Evidently, the systematic research on application of the PI(D) controllers under various conditions of uncertainty contributes to this mosaic.

Obviously, the stability is the first and most critical requirement of all control applications. However, the real-life control circumstances differ from the ideal nominal ones and so the uncertainty of the mathematical models has to be frequently taken into considerations. The attention of many researchers has been focused on investigation of robust stability for systems with parametric uncertainty – see e.g. (Barmish, 1994), (Bhattacharyya *et al.*, 1995), (Matušů and Prokop, 2011), (Aguirre and Suárez, 2006). Typical problem of practical PI(D) controller design is to ensure, that the calculated controller will guarantee stability not only for one assumed nominal controlled system, but also for the whole family of systems described by a model with parametric uncertainty. Such closed-loop control system is called as "robustly stable" and the controller itself is then robustly stabilizing one.

An array of techniques for calculation of (nominally) stabilizing PI and PID controllers have been already published, such as rules presented in (Söylemez *et al.*, 2003), the Tan's method described in (Tan and Kaya, 2003), (Tan *et al.*, 2006) or the Kronecker summation method from (Fang *et al.*, 2009). Furthermore, these methods have been also extended for robust stabilization of interval plants by their combination with the sixteen plant theorem (Barmish, 1994), (Barmish *et al.*, 1992). Nevertheless, this extension works only for PI but not for PID controllers.

The main aim of this paper is to present a method for computation of all possible robustly stabilizing PID controllers for interval plants and to demonstrate its serviceability by robust stabilization of an oblique wing aircraft model. More specifically, the goal is to refine the elegant and effective Tan's method (Tan and Kaya, 2003), (Tan *et al.*, 2006) by the ideas from (Ho *et al.*, 2001), (Ho *et al.*, 1998) and to make it applicable for computation of robustly stabilizing PID controllers. Previously, the computation of all (nominally) stabilizing PI or PID controllers, robustly stabilizing PI controllers and consequent choice of the specific controller with desired performance on the basis of the desired model method (formerly known as dynamics inversion method) (Vítečková, 2000) is shown in (Matušů, 2011). Then, the application of Kronecker summation method (Fang *et al.*, 2009) to robust stabilization of a chemical reactor or robust stabilization of a third order nonlinear electronic model is given in (Matušů *et al.*, 2011) or (Matušů *et al.*, 2010a), respectively. The robust stabilization of the same nonlinear electronic plant using the Tan's method (Tan and Kaya, 2003), (Tan *et al.*, 2006) is presented e.g. in (Matušů *et al.*, 2010b).

2. NOMINAL STABILIZATION

2.1 PI Control

First, the fundamentals relating to computation of stability regions for PI controllers are going to be summarized.

Suppose the classical closed-loop control system with a controller $C(s)$ and a controlled plant $G(s)$. The controller is assumed in the PI form:

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$$C(s)=k_P+\frac{k_I}{s}=\frac{k_Ps+k_I}{s} \tag{1}$$

The principal task is to determine its parameters k_P, k_I which guarantee stabilization of the controlled plant:

$$G(s)=\frac{B(s)}{A(s)} \tag{2}$$

Several effective methods for computation of stabilizing PI controllers have been already published – e.g. (Söylemez *et al.*, 2003), (Tan and Kaya, 2003), (Tan *et al.*, 2006), (Fang *et al.*, 2009). Here, the Tan's method from (Tan and Kaya, 2003), (Tan *et al.*, 2006) will be revisited and extended. This graphical approach is based on plotting the stability boundary locus. The substitution of s for $j\omega$ in the plant transfer function (2) and subsequent decomposition of the numerator and denominator into their even and odd parts result in:

$$G(j\omega)=\frac{B_E(-\omega^2)+j\omega B_O(-\omega^2)}{A_E(-\omega^2)+j\omega A_O(-\omega^2)} \tag{3}$$

Further, expressing the closed-loop characteristic polynomial and equating both real and imaginary parts to zero lead to the relations for the proportional and integral gains k_P, k_I:

$$\begin{aligned} k_P(\omega)&=\frac{P_5(\omega)P_4(\omega)-P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)}\\ k_I(\omega)&=\frac{P_6(\omega)P_1(\omega)-P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)} \end{aligned} \tag{4}$$

where

$$\begin{aligned} P_1(\omega)&=-\omega^2B_O(-\omega^2)\\ P_2(\omega)&=B_E(-\omega^2)\\ P_3(\omega)&=\omega B_E(-\omega^2)\\ P_4(\omega)&=\omega B_O(-\omega^2)\\ P_5(\omega)&=\omega^2A_O(-\omega^2)\\ P_6(\omega)&=-\omega A_E(-\omega^2) \end{aligned} \tag{5}$$

Simultaneous calculations of the equations (4) for suitable range of ω and plotting the obtained values into the (k_P,k_I) plane determine the stability boundary locus. The obtained curve together with the line $k_I=0$ split the (k_P,k_I) plane into the stable and unstable regions. The decision if the respective region represents stabilizing or unstabilizing area can be done simply using a test point within each region. Nonetheless, the appropriate frequency gridding could represent a potential problem. Thus, the Nyquist plot based technique from (Söylemez *et al.*, 2003) can be used for improvement of the method. In this improvement, the frequency ω can be separated into several intervals within which the stability or instability can not change. The borders of such intervals are defined by the real values of ω which fulfill the equation:

$$\mathrm{Im}\left[G(s)\right]=0 \tag{6}$$

The obtained intervals could be helpful for the proper frequency scaling.

2.2 PID Control

Now, the issue of feedback stabilization will be elaborated again, but for the case of ideal PID controller given by:

$$C(s)=k_P+\frac{k_I}{s}+k_Ds=\frac{k_Ps+k_I+k_Ds^2}{s} \tag{7}$$

The principal idea for obtaining the relevant stability regions is to fix one controller parameter to a certain value and calculate the stability boundary locus using two remaining parameters analogous to the procedure presented in the previous subsection 2.1.

The expression for the stability boundary locus in the (k_P,k_I) plane for a fixed value of k_D leads to a bit modified equations for proportional and integral gains:

$$\begin{aligned} k_P(\omega,k_D)&=\frac{P_5(\omega)P_4(\omega)-P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)}\\ k_I(\omega,k_D)&=\frac{P_6(\omega)P_1(\omega)-P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)} \end{aligned} \tag{8}$$

where

$$\begin{aligned} P_1(\omega)&=-\omega^2B_O(-\omega^2)\\ P_2(\omega)&=B_E(-\omega^2)\\ P_3(\omega)&=\omega B_E(-\omega^2)\\ P_4(\omega)&=\omega B_O(-\omega^2)\\ P_5(\omega)&=\omega^2A_O(-\omega^2)+\omega^2B_E(-\omega^2)k_D\\ P_6(\omega)&=-\omega A_E(-\omega^2)+\omega^3B_O(-\omega^2)k_D \end{aligned} \tag{9}$$

Note that the last two terms in (9) depend on derivative constant k_D. From the viewpoint of practical computation, k_D is considered to be chosen and corresponding set of boundary parameters k_P, k_I is consequently calculated while this process is repeated for several selected values of k_D. Thus, the final stability regions are successively plotted through the "(k_P,k_I) sections" in the (k_P,k_I,k_D) space.

Alternatively, the stability boundary locus in the (k_P,k_D) plane for a fixed value of k_I can be computed. This scenario would change the equations (8) and (9) to, respectively:

$$\begin{aligned} k_P(\omega,k_I)&=\frac{P_5(\omega)P_4(\omega)-P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)}\\ k_D(\omega,k_I)&=\frac{P_6(\omega)P_1(\omega)-P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)} \end{aligned} \tag{10}$$

where