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Robust non-fragile fractional order PID controller for linear time invariant fractional delay systems

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A fractional order PID controller is designed to stabilize fractional delay systems with commensurate orders and multiple commensurate delays, where the time delays in the system may belong to several distinct intervals. Moreover, the controller parameters should belong to given intervals. In order to stabilize the system, the D-subdivision method is employed to choose the stabilizing set of the controller parameters from their available values. Furthermore, the nearest values of the obtained stabilizing set to their mean values are selected as the controller parameters so that a non-fragile controller is concluded. Two numerical examples evaluate the proposed control design method.

1. Introduction

Fractional-order time-delay differential systems (FOTDSs) have been given considerable attention in the last decade due to their properties and several potential applications [1]. Since time-delay systems (TDSs) and fractional-order systems (FOSs) provide a useful tool to characterize the hereditary and memory properties, FOTDSs have been successfully applied to model different types of the complex physical phenomena [2–5]. In addition to appearing in the modeling applications, the expected closed-loop results encourage the establishment of a set of fractional-order control strategies for TDS. The response of the closed-loop system by using the fractional-order controllers can be much more efficient than the traditional controllers with integer order [6]. Furthermore, fractional-order controllers guarantee some required properties of the desired closed-loop responses [7,8]. The proportional (P), the fractional-order proportional-integral (FOPI), the fractional-order proportional-derivative (FOPD), and the fractional-order proportional-integral-derivative (FOPID) controllers have been commonly used in many industrial applications [6,7]. Owing to their wide applicability in the industrial world, many approaches have been proposed to design FOPID controllers for fractional delay systems as well.

The FOPID controllers have been developed for a given FOS plus a time delay to achieve the desired gain and phase margins [9]. The FOPD controller has been designed for a rational transfer function plus a time delay such that the closed-loop system is H_∞ robust stable with the uncertainty of the gain in the model [10]. Recently, by using a graphical algorithm and an optimization of the fractional orders of the controller, respectively, the parameters and the fractional orders of an FOPID controller have been calculated to stabilize a rational transfer function plus a time delay and to obtain greater stabilizing region [11].

The first order plus time delay system, as a simple case of the fractional delay systems, has been employed in several industrial case studies. An FOPI controller has been designed to achieve a set of frequency-domain specifications for a stable first order plus time delay system in [12]. In this regard, regions of feasible frequency specifications for several forms of the FOPI controller have been considered to control a first order plus time delay system and the obtained results have been compared to traditional PI controllers [13]. Moreover, by applying the optimal PID and FOPID, the integrated absolute error has been minimized subjected to a constraint on maximum sensitivity value for the first order plus time delay system [14]. For a class of first order FOS with integrator plus time delay, an FO-[PD] controller has been designed to satisfy the

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desired gain crossover frequency and phase margin [15]. In another study, parameters of the same control structure have been uniquely determined in order to satisfy the same frequency specifications along with the flatness of the phase plot around the cut off frequency [16]. This extra feature increases the closed-loop robustness against model and process mismatches. In [17], an FOPI controller has been designed for a class of single fractional order pole systems with constant time delay and applied to control a heat flow platform.

A closed-loop system may be destabilized by small perturbations in the values of the designed controller parameters. To consider this deficiency, the center of gravity of the admissible region of controller parameters has been chosen to design a non-fragile FOPD controller for integral plus time delay systems in [18].

In some modeling applications, the constant time delay has been assumed to belong to an interval. For instance, the irrigation canals are distributed over long distances with significant large time delays (between the water resources and the water consumers). In addition, their dynamics change in accordance with the variation of operating conditions [19–21]. In this respect, robust FOPID controllers have been developed by fitting the system Bode envelopes to a first order plus time delay system, while it is assumed that the time delay belongs to a bounded interval [22,23].

In [24], a stabilizing PID controller has been proposed for a given FOS with one unstable pole and a time delay where the time delay belongs to an interval starting at zero. In some practical systems, the lower bound of the time delay is greater than zero [20,25,26]. In many medical and biological systems, the time delay belongs to several distinct intervals due to their behavior dependencies on the amount of a given drug, the age, and the gender of patients [27]. As another practical example, drivers have a delay in brake reaction time, which depends on drivers' state of awareness and psychological and physiological states. It has been shown that the delay belongs to several distinct intervals such that the lower bound of the intervals is greater than zero [28]. The existing methods of designing a stabilizing FOPID controller for such kinds of systems can be used if it is assumed that the time delay belongs to an interval which is the union of all distinct intervals. In this way, an insufficient constraint on the desired intervals of the time delay is added by this assumption. Therefore, the space of admissible control parameters can become smaller or even vanish.

In the present study, a FOPID controller is designed to stabilize commensurate fractional order linear time invariant systems with multiple commensurate delays, belonging to several distinct intervals. The first step in the proposed designing method is that for each point in the available space of the controller parameters, the stability of the closed-loop system is determined by using the D-subdivision method, which has been extended for the FOTDS [29]. In addition, by virtue of many physical problems in controller implementation, the uncertainty may occur in the control parameters. Thus, as the second step, the non-fragile controller is concluded when the controller parameters are chosen as the center of the acceptable set, which are obtained herein for the controller parameters.

The remainder of the paper is structured as follows. Outlines of the mathematical formulation and definitions are presented in Section 2. Section 3 contains the main results. In the first step, the controller parameters are calculated such that the closed-loop system becomes stable for any value of time delay belonging to the given intervals. Secondly, the non-fragile controller is determined by introducing the related formulas. Section 4 provides two numerical examples and finally, the paper is concluded in Section 5.

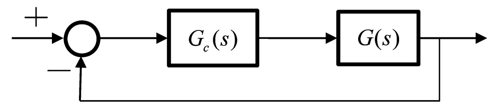

Fig. 1. Feedback control system.

2. Preliminaries and definitions

It is assumed that the system has the following transfer function

$$G(s)=\frac{Q(s)}{P(s)}=\frac{s^{\frac{N}{K}}+\sum_{m=0}^{N-1}\left(\sum_{k=0}^{l_N} b_{m,k}e^{-k\tau s}\right)s^{\frac{m}{K}}}{s^{\frac{D}{K}}+\sum_{m=0}^{D-1}\left(\sum_{k=0}^{l_D} a_{m,k}e^{-k\tau s}\right)s^{\frac{m}{K}}}, \tag{1}$$

where $D, N, K, l_N, \quad l_D \in \mathbb{N}, N \leq D$ and $a_{m,k}, \quad b_{m,k} \in \mathbb{R}$.

Assumption 1. Polynomials $Q(s)$ and $P(s)$ are coprime.

Remark 1. $P(s)$ is a multi-valued function and has an infinite number of branches because of the presence of non-integer powers of s. However, the Hurwitz stability of $P(s)$ is resulted from its zeros on the first Riemann sheet. Therefore, the principal branch of $P(s)$ is considered for the stability analysis.

The principal branch of $P(s)$, $P_{p_b}(s): \mathbb{C} \to \mathbb{C}$, is defined as follows

$$P_{p_b}(s)=\begin{cases} |s|^{\frac{D}{K}}e^{j\frac{D}{K}arg(s)}+\sum_{m=0}^{D-1}\sum_{k=0}^{l_D} a_{m,k}e^{-k\tau s}|s|^{\frac{m}{K}}e^{j\frac{m}{K}arg(s)} & \text{if} \quad s\neq 0 \\ \sum_{k=0}^{l_D} a_{0,k} & \text{if} \quad s=0 \end{cases}, \tag{2}$$

where j and $arg(s) \in (-\pi, \pi]$ denote $\sqrt{-1}$ and the principal value of the argument of s for $s \neq 0$ and a given $\tau \geq 0$. According to (2), P_{p_b} is a single valued function of s.

The generalized scalar FOTDS corresponds to system (1) is written as

$$\frac{d^{D/M}y(t)}{dt^{D/M}}+\sum_{m=0}^{D-1}\left(\sum_{k=0}^{l_D} a_{m,k}\frac{d^{m/K}y(t-k\tau)}{dt^{m/K}}\right)$$
$$=\frac{d^{N/K}u(t)}{dt^{N/K}}+\sum_{m=0}^{N-1}\left(\sum_{k=0}^{l_N} b_{m,k}\frac{d^{m/K}y(t-k\tau)}{dt^{m/K}}\right), \tag{3}$$

where $u(t)$ and $y(t)$ are respectively the input and the output of the system.

3. Tuning of the fractional order controller

The closed-loop feedback control system is shown in Fig. 1. $G_c(s)$ is the following fractional order controller.

$$G_c(s)=\frac{k_p}{s^{\frac{N_p}{D_p}}}+\frac{k_i}{s^{\frac{N_i}{D_i}}}+k_d s^{\frac{N_d}{D_d}}, \tag{4}$$

where $k_p, k_i, k_d \in \mathbb{R}$, $N_p, N_i, N_d, D_p, D_i, D_d \in \mathbb{N}$ such that $N_i < 2D_i$, $N_d < 2D_d$ and $N_p < D_p$.

Assumption 2. Throughout this paper, it is assumed, without loss of generality, that $N_p/D_p < N_i/D_i$.

Remark 2. The reason for using controller (4) is that this controller is the extension of the family of FOPID and PID controllers. The FOPID controller can be resulted by assuming $N_p = 0$, $N_i < 2D_i$, and $N_d < 2D_d$. Controller (4) reduces to a PID controller by considering