Journal of Process Control 21 (2011) 17–27

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

PID controller frequency-domain tuning for stable, integrating and unstable processes, including dead-time

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ARTICLE INFO

Article history:
Received 25 May 2010
Received in revised form 23 August 2010
Accepted 3 September 2010

Keywords:
PID controller tuning
PI controller tuning
Robustness
Measurement noise

ABSTRACT

In the present paper a new tuning procedure is proposed for the ideal PID controller in series with the first-order noise filter. It is based on the recently proposed extension of the Ziegler–Nichols frequency-domain dynamics characterization of a process $G_p(s)$. Measured process characteristics are the ultimate frequency and ultimate gain, the angle of the tangent to the Nyquist curve of the process at the ultimate frequency, and $G_p(0)$. For a large class of processes the same tuning formulae can be effectively applied to obtain closed-loop responses with predictable properties. Load disturbance step responses without the undershoot and reference step responses with negligible overshoot are obtained by analyzing a test batch consisting of stable, integrating and unstable processes, including dead-time and oscillatory dynamics. The proposed tuning makes possible to specify the desired sensitivity to the high frequency measurement noise and the desired maximum sensitivity. Comparison with the optimal ideal PID controller in series with the first-order noise filter is presented and discussed. The extension of the proposed method to the PI controller tuning is direct. Comparison with the optimal PI controller is presented and discussed.

1. Introduction

Design of the PID controller based on application of tuning formulae is initiated by Ziegler and Nichols [1] and still predominates over the optimization methods [2–5]. The basic idea of this approach is to find some transformations of a small amount of measured process characteristics into the values of the PID gains. For example, a refinement of the Ziegler–Nichols tuning formulae, proposed in [6], is based on estimation of the ultimate frequency ω_u, ultimate gain k_u and the gain $G_p(0)$ of a process $G_p(s)$. The same measured data are used in [7,8]. Time domain process dynamics characterization and PID controller tuning is initiated also by Ziegler and Nichols [1], used in [3,7,9] and developed further through the IMC-PID design and lambda-tuning in [10,11]. A detailed overview of tuning formulae developed until now is presented in [12]. Based on the frequency or time domain process dynamics characterization, they are developed to cover some specific process dynamic characteristics. Different tuning procedures are derived for stable, integrating and unstable processes.

In the present paper a unified PID controller tuning is proposed for a large class of processes (stable, integrating and unstable, including dead-time and oscillatory dynamics) satisfying condition that the limit cycle exists, defined by k_u and ω_u. Then, it is extended to the PI controller tuning.

According to [12] the greatest number of tuning formulae is derived for the ideal PID controller. The same holds true for the most of tuning formulae mentioned above. This means that the rules are derived for adjusting the proportional gain k, integral time T_i and derivative time T_d, taking the derivative (noise) filter time constant T_f equal to zero. Then, T_f can be determined as some fraction of the derivative time T_d, for example as $T_f = T_d/N$, $N = 2$–10, resulting into deterioration of the performance/robustness tradeoff, obtained by the tuning rule proposed for $N = \infty$. Since in industry applications T_f must satisfy relation $T_f > 0$, the derivative (noise) filter must be an integral part of the PID optimization and tuning procedures [13]. In the present paper the ideal parallel PID controller in series with the noise filter $F_{\mathrm{NF}}(s) = 1/(T_f s + 1)$, given by

$$C_{\mathrm{PID}}(s) = \frac{k_d s^2 + ks + k_i}{s(T_f s + 1)} \tag{1}$$

is used, where the integral and derivative gains are related with the proportional gain k by the relation $k_i = k/T_i$ and $k_d = kT_d$, respectively. Tuning method derived here, as well as optimization procedure used for comparison [5], include adjustment of the four parameters k, k_i, k_d, T_f. When these parameters are determined, and controller $C(s)$ in Fig. 1 is realized as PID controller (1), then $G_{ff}(s)$ can be designed and tuned as in [2]. Also, when k, k_i, k_d and T_f are determined, $C_{\mathrm{PID}}(s)$ can be implemented as in (1) or in the traditional form, where noise filtering affect the derivative term only [4].

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doi:10.1016/j.jprocont.2010.09.007

Recently, it has been shown in [5] that an additional parameter in the frequency domain defines an extension of the Ziegler–Nichols approach to the process dynamics characterization, which makes possible to capture the essential dynamic characteristics of a large class of stable, oscillatory, integrating and unstable processes, including dead-time. This additional parameter φ is the angle of the tangent to the Nyquist curve $G_p(i\omega)$ at the ultimate frequency ω_u. It is demonstrated that practically the same performance/robustness tradeoff is obtained by applying max(k) PID controller optimization [5] with exact $G_p(i\omega)$ or with approximation $G_m(i\omega)$, defined by the measured process characteristics ω_u, k_u, φ, $G_p(0)$ and

$$G_m(s) = \frac{A\omega_u e^{-\tau s}}{s^2+\omega_u^2 - A\omega_u e^{-\tau s}}\frac{1}{k_u}, \quad A = \frac{\omega_u k_u G_p(0)}{1+k_u G_p(0)}, \quad \tau = \frac{\varphi}{\omega_u}. \tag{2}$$

In Sections 2 and 3, transformations of the measured process characteristics ω_u, k_u, φ, $G_p(0)$ into the four parameters k, k_i, k_d, T_f of the PID controller (1) are derived. Analysis and evaluation of the proposed PID controller tuning is presented in Section 4. It will be shown that the same tuning formulae can be applied to a large class of stable, integrating and unstable processes, including dead-time and oscillatory dynamics. Moreover, the proposed PID controller tuning gives closed-loop system load disturbance and reference responses with predictable properties. The proposed tuning method offers the possibility to specify the desired values of the sensitivity to high frequency measurement noise M_n and maximum sensitivity M_s. Extension of the proposed method to the PI controller tuning, with the specified value of M_s, is presented in Section 5.

2. Problem statement

Tuning formulae for the controller $C(s)$ in Fig. 1, realized as the PID controller (1), are derived by analyzing a virtual control system presented in Fig. 2. Controller $C^*(s)$ in Fig. 2a is used to stabilize the oscillatory process

$$G_p^*(s) = \frac{k_u G_p(s)}{1+k_u G_p(s)}. \tag{3}$$

When the oscillatory process (3) is approximated by

$$G_p^*(s) = \frac{A\omega_u e^{-\tau s}}{s^2+\omega_u^2}, \tag{4}$$

as in [5], one obtains that the controller $C^*(s)$ is used to stabilize the oscillatory process (4), as in Fig. 2b. Approximation (2) is obtained from (3) to (4), for $G_p(s) \approx G_m(s)$ [5]. Parameter A and dead-time τ in (4) are defined as in (2), by the measured process characteristics ω_u, k_u, φ, $G_p(0)$. This is further discussed in the two Appendices.

Most of the PID controllers operate as regulators [14] and rejection of the step load disturbance is of primary importance to evaluate their performance [15]. Thus, equivalency of the controller $C^*(s)$ in Fig. 2 and controller $C(s)$ in Fig. 1 is obtained from equivalency of the load disturbance step responses of the closed-loop systems presented in Fig. 2 and Fig. 1. Load disturbance step responses of the closed-loop systems in Fig. 2a and b, for

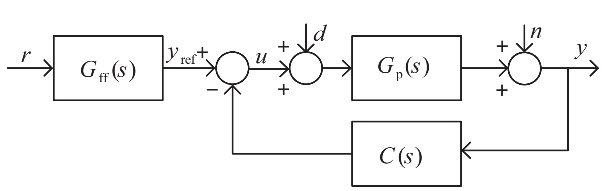

Fig. 1. Process $G_p(s)$ with the controller $C(s)$.

$Y_{ref}(s) = D(s) = 0$, $D^*(s) = 1/s$, are given by

$$Y_d^*(s) = \frac{G_p^*(s)}{1+C^*(s)G_p^*(s)}\frac{1}{s} = \frac{k_u G_p(s)}{1+k_u(1+C^*(s))G_p(s)}\frac{1}{s}. \tag{5}$$

From Fig. 1, for $Y_{ref}(s) = 0$, $D(s) = 1/s$, and Fig. 2a, for $Y_{ref}(s) = D^*(s) = 0$, $D(s) = 1/s$, it follows

$$Y_d(s) = \frac{G_p(s)}{1+C(s)G_p(s)}\frac{1}{s} = \frac{G_p(s)}{1+k_u(1+C^*(s))G_p(s)}\frac{1}{s}. \tag{6}$$

Then, from (5) to (6) one obtains that equivalency of the closed-loop systems in Figs. 1 and 2b is defined by

$$Y_d(s) = \frac{1}{k_u}Y_d^*(s), \tag{7}$$

$$C(s) = k_u(1+C^*(s)). \tag{8}$$

Controller $C^*(s)$ is defined in Section 3. Then, tuning formulae are derived for implementing the controller $C(s)$ in Fig. 1, given by (8), as the PID controller (1).

3. Tuning of the ideal PID controller in series with the first order noise filter

The load disturbance response of the closed-loop system in Fig. 2b is defined by

$$Y_d^*(s) = G_d^*(s)D^*(s), \quad Y_{ref}(s) = 0, \quad D^*(s) = \frac{1}{s}, \tag{9}$$

$$G_d^*(s) = \frac{G_p^*(s)}{1+C^*(s)G_p^*(s)}. \tag{10}$$

The complementary sensitivity function of the closed-loop system in Fig. 2b is given by $T^*(s) = L^*(s)/(1+L^*(s))$, where $L^*(s) = C^*(s)G_p^*(s)$ is the loop transfer function. Let the complementary sensitivity function $T^*(s)$ be defined by

$$T^*(s) = \frac{N(s)e^{-\tau s}}{P^2(s)}, \quad N(s) = \eta_2 s^2 + \eta_1 s + 1, \quad P(s) = \lambda^2 s^2 + 2\zeta\lambda s + 1, \tag{11}$$

where time constant $\lambda > 0$, damping ratio $\zeta > 0$, η_1 and η_2 are free parameters which will be determined to obtain the desired dynamic characteristics of the closed-loop system. Then, from $L^*(s) = N(s)e^{-\tau s}/(P^2(s) - e^{-\tau s}N(s))$, for $G_p^*(s)$ defined by (4), one obtains the controller $C^*(s)$ in the form

$$C^*(s) = \frac{1}{A\omega_u}\frac{(s^2+\omega_u^2)(\eta_2 s^2 + \eta_1 s + 1)}{(\lambda^2 s^2 + 2\zeta\lambda s + 1)^2 - e^{-\tau s}(\eta_2 s^2 + \eta_1 s + 1)}. \tag{12}$$

For $G_p^*(s)$ defined by (4), from (10) and (12) one obtains that the load disturbance transfer function $G_d^*(s)$ is defined by

$$G_d^*(s) = \frac{(\lambda^2 s^2 + 2\zeta\lambda s + 1)^2 - e^{-\tau s}(\eta_2 s^2 + \eta_1 s + 1)}{s^2+\omega_u^2}\frac{A\omega_u e^{-\tau s}}{(\lambda^2 s^2 + 2\zeta\lambda s + 1)^2},$$
$$\lambda > 0, \quad \zeta > 0. \tag{13}$$

To avoid oscillatory load disturbance response, free parameters η_1 and η_2 are determined to satisfy the condition

$$\left((\lambda^2 s^2 + 2\zeta\lambda s + 1)^2 - e^{-\tau s}(\eta_2 s^2 + \eta_1 s + 1)\right)\Big|_{s=\pm i\omega_u} = 0, \tag{14}$$

in order to cancel the pair of complex-conjugate poles $s_{1,2} = \pm i\omega_u$ in (13). These values of η_1 and η_2 are given by

$$\eta_1 = \frac{\alpha_1 \sin(\omega_u\tau) + \alpha_2\cos(\omega_u\tau)}{\omega_u}, \quad \eta_2 = \frac{\alpha_2\sin(\omega_u\tau) - \alpha_1\cos(\omega_u\tau) + 1}{\omega_u^2}, \tag{15}$$

$$\alpha_1 = \lambda^4\omega_u^4 - 2\lambda^2\omega_u^2(1+2\zeta^2) + 1, \quad \alpha_2 = 4\zeta\lambda\omega_u(1-\lambda^2\omega_u^2). \tag{16}$$