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Stabilization and algorithm of integrator plus dead-time process using PID controller

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ABSTRACT

In this paper, we develop an algorithm to determine the entire set of stabilising PID parameters for the integrator plus dead-time process. Our method is a combination of the traditional D-partition technique and graphical technique. We first apply the D-partition technique to address the stability regions in the plane of proportional and integral gains, which depend on the steady-state gain and the time delay. Next for fixed proportional gain, again using the D-partition technique we obtain a series of critical straight lines in the plane of integral and derivative gains and then choosing the derivative gain as a parameter, we investigate how the derivative gain affects the distribution of the roots of the corresponding characteristic equation of the closed-loop control system, which provide the stability regions in the plane of integral and derivative gains. On the base of the results obtained in the previous two steps, an efficient algorithm for determining the entire set of stabilising PID parameters is also proposed. Finally, numerical simulations are given to support our theoretical results.

1. Introduction

The proportional-integral-derivative (PID) controller and its variations are generic control loop feedback mechanisms widely used in industrial control systems. It has been stated, for example, that 98% of control loops in the pulp and paper industries are controlled by PI controllers [1] and that, in process control applications, more than 95% of the controllers are of PID type [2]. The primary problem associated with the use of PI or PID controllers is tuning, that is, the determination of PID controller parameters to produce satisfactory control performance. There has been a great amount of research work on the tuning of PI (proportional integral), PID and lag/lead controllers (see, e.g., [3,4] and references therein). It is well known that making the closed-loop system stable is a primary requirement of the tuning of PID type controllers. So, it is often desired to construct the complete set of stabilising PID parameters. With the complete set of stabilising PID controller parameters being available for a given process, it can avoid the time-consuming stability check for each set of PID controller parameters in the searching process and thereby to save the controller tuning time. Unfortunately, most of the industrial PID designs are still carried out using only empirical techniques such as the Ziegler–Nichols rules [5,6] and the construction of the complete set of stabilising PID controller parameters is not a trivial task, especially for the process with a time delay since the corresponding characteristic equation is a quasi-polynomial with an infinite number of poles which makes the stability analysis extremely difficult.

Time delay is ubiquitous in chemical, biological, and process control and unstable process with time delay makes control system design a difficult task, which has attracted increased attention from control community [7]. Recently, theoretically obtaining the set of stabilising PID controller has become an increasing activity and interest on the study of process control. Using root locus method, Huang and Chen [8] studied the issue of stabilizing a time delayed unstable process and showed that the normalized time delay should be less than 1 for P/PI controller to stabilize the first-order delayed unstable process, while it should be less than 2 for PD/PID controller. In [9], the authors applied D-partition method to characterize the stability domain in the space of system and controller parameters. The stability boundary is reduced to a transcendental equation, and the whole stability domain is drawn in two-dimensional plane by sweeping the remaining parameters. However, this result only provides sufficient condition regarding the size of the time delay for stabilization of first-order unstable processes. Using Nyquist criterion, Xiang et al. [10] investigated the stabilization of second-order unstable delay processes by PID controllers and obtained the necessary and sufficient condition concerning the maximal delay for stabilizability. In [11,12], two cascaded PID control loops are used to maintain the gas injection in the tubing constant and control gas-lifted wells in the context of instable flows and using a similar D-partition method the authors investigated the impact of the time delay on the stability of the trivial solution of the saturation-free model. However, the complete stability region in the literature mentioned above remains

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unknown, which is important to the performance design of the processes.

Recently, a method based on a version of Hermite–Biehler theorem applicable to quasi-polynomials has been used to determine the complete set of stabilising PID controller parameters for first-order time delay system by Silva et al. [13–15] and for second-order time-delay systems by Ou et al. [16]. But this approach is mathematically involved and the maximal stabilizable time delay for some typical yet simple processes still remains obscure. Using the classical Nyquist stability criterion, Martelli [17] obtained the same results as those in [15] in a simpler way. Using a parameter space approach combined with the generalized Hermite–Biehler theorem, Wang [18,19] studied the stabilizing regions of PID controllers for second-order and nth order time delay systems. However, the results obtained in the above literature aren't applicable to the integrator plus dead-time processes (IPDT) with the transfer function

$$G(s) = \frac{ke^{-Ls}}{s}, \tag{1}$$

where $k > 0$ represents the steady-state gain of the plant, $L \geqslant 0$ is the dead time. In this paper, we focus on the case when the controller is of the PID type, i.e.,

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{2}$$

The process (1) was first proposed by Chien and Fruehauf [20], who suggested that many chemical processes can be modelled for the purpose of feedback controller tuning by a transfer function (1). This type of model is able to adequately represent the dynamics of many processes over the frequency range of interest for feedback controller design, i.e., near the ultimate frequency where the total open-loop Nyquist plot approaches the $(-1,0)$ point. Friman and Waller [21] demonstrated that this type of model can also be used in multivariable systems. Tyreus and Luyben [22] presented tuning rules that give the optimal reset time and controller gain for PI control of this type of process. Luyben [23] extended the previous work with PI control to PID controllers, where frequency domain methods are used to show that the derivative tuning constant should be set equal to the reciprocal of the ultimate frequency. A systematic procedure for identifying the transfer-function parameters for IPDT and inverse-response processes was presented in [24,25] where a PI controller tuning rule was synthesized using +2DB maximum closed-loop log modulus criteria. More recently, Panda et al. [26] studied the set-point changes and load disturbances for this type of process IPDT using a novel control scheme, which is proposed by integrating Smith predictor and a PID controller. However, the theoretical results on the entire set of stabilizing PID parameters for the process (1) still remain unknown. In the present paper, the main objective is to determine the parameter set (k_p, k_i, k_d) of the PID controller such that the feedback closed-loop process is stable. We present a novel approach to derive analytical expressions for describing the boundaries of the stability domain in the space of PID controller parameters. These expressions can be used to construct the complete set of stabilising PID controller parameters. We would also like to mention that the similar issues we study here were also addressed in [15,18,28,27]. The authors focused on the processes without time delay in [28,27], while in [15,18] the authors focused on the processes with time delay. Based on the generalization of the Hermite–Biehler theorem for complex polynomials with complex leading coefficients, Ho [27] investigated the problem of characterizing all admissible PID controllers and shown that for a fixed proportional gain, the set of admissible integral and derivative gains lie in a union of convex sets. In terms of plotting the stability boundary locus, Tan et al. [28]presented the boundaries of the limiting values of PI and PID controller parameters that guarantee stability and found the stabilizing region of PI parameters for the control of a plant with uncertain parameters. In [15], the authors employed a version of the Hermite–Biehler theorem applicable to quasi-polynomials to investigate the complete set of stabilizing PID parameters for the first-order process with a time delay $G(s) = ke^{-Ls}/(1+Ts)$ and showed that for each admissible proportional gain the stabilizing set in the plane of the integral and derivative gains is either a trapezoid, a triangle or a quadrilateral. However, as you will see below, the stabilizing set for (1) in the plane of the integral and derivative gains is always a quadrilateral for each admissible proportional gain. In [18], the author considered the same topic as [15] and extended the results to a second-order process with a time delay via the graphical stability criterion. In the present paper, we combine the D-partition and graphical technique to study the problem of stabilizing a integrator plus dead-time process using a PID controller. We first determine the stability region in the plane of proportional and integral gains. The area of such stability region is decreased with increasing the time delay. Then for fixed proportional gain, taking the derivative gain as a parameter, we investigate how the derivative gain affects the distribution of the roots of the corresponding characteristic equation of the closed-loop control system and obtain the stabilizing set in the space of the integral and derivative gains. Compared with the methods used in [15,18], the advantage of our method is that the mechanism of how the time delay and derivative gain affect the stability of closed-loop system is much clearer. In addition, the issue of how the time delay affects the stability region in the plane of proportional and integral gains was not investigated in [27,15,18], and hence addressing this issue is one of the contributions of our work in this paper.

The rest of the paper is organized as follows: in Section 2, we first compute the range of admissible proportional and integral gains. In Section 3, for fixed value of proportional gain, we analytically compute the critical curves on the integral-derivative plane and investigate the changes of the positive real parts of the roots of the corresponding characteristic equation, which allow us to obtain the stabilizing region in the integral-derivative plane. The procedures for determining the complete sets of stabilising PID controllers and some numerical simulations are addressed in Section 4. Finally, we conclude the paper in Section 5.

2. Stabilization using PI controller

The closed-loop stability is equivalent to the condition that all the roots of the characteristic equation lie in the open left-half plane. The characteristic equation of the closed-loop process is

$$\delta_1(s) = s^2 + k(k_i + k_p s + k_d s^2)e^{-Ls} = 0. \tag{3}$$

Due to the presence of the exponential term, the number of the roots of the characteristic Eq. (3) may be infinite, which makes the problem of analyzing the closed-loop system a difficult one. Here we will employ the so called D-partition technique to attain our aim. For D-partition technique, refer to the Appendix A.

We first study the case of the stabilization of the process under the PI controller, i.e, $k_d = 0$. In this case, the characteristic equation of the closed-loop system becomes

$$\delta_2(s) = s^2 + k(k_i + k_p s)e^{-Ls} = 0. \tag{4}$$

Starting with the delay-free system, i.e., $L = 0$, Eq. (4) becomes

$$s^2 + kk_p s + kk_i = 0. \tag{5}$$

Since this is a second-order polynomial, the closed-loop stability is equivalent to all the coefficients having the same sign. So, we have the stabilizing set of PI for the delay-free plants as follows