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# Thermal response simulation for tuning PID controllers in a 1016 mm guarded hot plate apparatus

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ABSTRACT

A mathematical model has been developed and used to simulate the controlled thermal performance of a large guarded hot-plate apparatus. This highly specialized apparatus comprises three interdependent components whose temperatures are closely controlled in order to measure the thermal conductivity of insulation materials. The simulation model was used to investigate control strategies and derive controller gain parameters that are directly transferable to the actual instrument. The simulations take orders-of-magnitude less time to carry out when compared to traditional tuning methods based on operating the actual apparatus.

The control system consists primarily of a PC-based PID control algorithm that regulates the output voltage of programmable power amplifiers. Feedback parameters in the form of controller gains are required for the three heating circuits. An objective is to determine an improved set of gains that meet temperature control criteria for testing insulation materials of interest.

The analytical model is based on aggregated thermal capacity representations of the primary components and includes the same control algorithm as used in the actual hot-plate apparatus. The model, accounting for both thermal characteristics and temperature control, was validated by comparisons with test data.

The tuning methodology used with the simulation model is described and results are presented. The resulting control algorithm and gain parameters have been used in the actual apparatus without modification during several years of testing materials over wide ranges of thermal conductivity, thickness, and insulation resistance values.

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## 1. Introduction

The National Institute of Standards and Technology (NIST) operates a 1016 mm guarded hot-plate (GHP) apparatus to establish Standard Reference Materials (SRMs) for the thermal insulation industry and research community. An overall view of the apparatus is shown in Fig. 1. These SRMs are recognized internationally and used primarily to calibrate in-house instruments for measuring the thermal conductivity or thermal resistance (*R*-value) of manufactured products, or of insulating materials under development.

The guarded hot-plate method, which is based on Fourier's law and described by the ASTM (2004) has been standardized by both the International Standards Organization (1991) and ASTM International [1,2].

From a control viewpoint, GHP instruments respond slowly and tuning criteria differ markedly from those for conventional industrial processes. For thermal insulation SRMs, the target expanded uncertainty in steady-state heat transmission properties is approximately 2%. The criteria used in the present investigation emphasize tight temperature control, especially during the final phase of test runs, rather than rapid response from a "cold" start. Typically, the duration of a test run is about a day from a "cold" start and steady-state data are based only on operation during the last 2–4 h. This ending interval is denoted as the "quasi-steady" phase.

The overall objective of the investigation is to validate a controller algorithm and settings for proportional, integral, and derivative gains for three apparatus controllers—potentially a total of nine interdependent parameters. The criterion for this set of gains is stable operation with excursions of temperatures and heating rates small enough to have a negligible effect on achieving the above accuracy target. Although not a primary emphasis, an important consideration is to minimize the time required to reach the phase where temperature and heat flow data become useful which, of course, reduces the overall test duration.

In this paper, a mathematical model is used to simulate control and derive gain parameters that can be transferred directly to

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**Nomenclature**

$A$ (m$^2$) Surface area
$c_p$ (J/(kg °C)) Specific heat
$C_i$ (J/°C) Thermal capacity, inner guard plate $= (mc_p)_i$
$C_m$ (J/°C) Thermal capacity, meter plate $= (mc_p)_m$
$C_o$ (J/°C) Thermal capacity, outer guard plate $= (mc_p)_o$
$G_{ga}$ (W/°C) Thermal conductance, outer guard to ambient air
$G_{gm}$ (W/°C) Thermal conductance across gap, inner guard to meter
$G_{ib}$ (W/°C) Thermal conductance, inner guard to bottom plate $= \lambda_2 A_i/L_2$
$G_{io}$ (W/°C) Thermal conductance, inner to outer guard $\approx \pi\lambda_g/\ln(d_o/d_i)$
$G_{is}$ (W/°C) Thermal conductance, inner guard to top plate $= \lambda_1 A_i/L_1$
$G_{mb}$ (W/°C) Thermal conductance, meter to bottom plate $= \lambda_2 A_m/L_2$
$G_{ms}$ (W/°C) Thermal conductance, meter to top plate $= \lambda_1 A_m/L_1$
$G_{os}$ (W/°C) Thermal conductance, outer guard to top plate $= \lambda_1 A_o/L_1$
$G_{ob}$ (W/°C) Thermal conductance, outer guard to bottom plate$= \lambda_2 A_o/L_2$
$d_i$ (m) Diameter, inner heater element, guard plate
$d_{io}$ (m) Partition diameter, inner–outer guard plate zones
$d_o$ (m) Diameter, outer heater element, guard plate
$e$ (°C, V) Error signal (set point-process variable)
$j$ Mathematical summation index
$k$ Discrete time index, $k \sim t/\Delta t_{sr}$
$K_i$ (V/°C s) Integral gain, positional algorithm
$K_p$ (V/°C) Proportional gain, positional algorithm
$K_d$ (V s/°C) Derivative gain, positional algorithm
$L$ (m) Thickness of test specimen
$m$ (kg) Mass
$q$ (W) Heating rate (electrical power)
$R_{pl}$ (Ω) Electrical resistance, power leads
$R_{tc}$ (Ω) Electrical resistance, total circuit
$R$-value (°C m$^2$/W) Thermal resistance
$t$ (s) Time
$T$ (°C) Temperature
$v$ (VDC) Controller output voltage
$v_{hl}$ (VDC) Controller output voltage, high-limit
$V$ (VDC) Voltage, PS output or heater element
$\Delta t$ (s) Numerical solution time step
$\Delta t_{sr}$ (s) Controller scan time step
$\kappa_P$ (V/°C s, V/V s) Proportional gain, incremental algorithm
$\kappa_D$ (V/°C, V/V) Derivative gain, incremental algorithm
$\lambda$ (W/(m °C)) (Apparent) Thermal conductivity, test specimen

*Additional subscripts*

$a$ Ambient air inside hot plate chamber
$b$ Bottom test specimen or cold plate (2)
$C$ Cold plate
$g$ Guard plate
$he$ Heater element
$H$ Hot plate
$i$ Inner guard zone
$m$ Meter section
$o$ Outer guard zone
$ps$ Power supply
$s$ Top test specimen or cold plate (1)

a highly specialized apparatus. The simulation model, comprising simple response expressions, is shown to give results that agree with data close enough to analytically investigate various controller settings. An important feature of the simulation method is that it takes an order of magnitude less time to carry out when compared to the traditional tuning method based on actual operation of this particular apparatus.

### 1.1. Principle of operation

The primary components of the present apparatus are three 1016 mm diameter aluminum plates, arranged schematically as illustrated in Fig. 2. The center hot plate consists of a guarded hot plate (meter plate), 405.6 mm in diameter by 16.13 mm thick, and surrounded by a heated (primary) guard plate, 1016 mm in diameter. The meter plate and guard plate are physically separated by a uniform 0.89 mm-thick air gap, measured at the plate surfaces. The cross-sectional profile of the gap is diamond shaped (Fig. 2) in order to minimize lateral heat flows across the gap as well as to facilitate assembly and installation of temperature sensors (described later). The meter plate is supported by three stainless steel pins, placed at angular intervals of 120° and pressed radially across the gap to maintain a uniform gap width. On either side of the center hot plate are two cold plates, 1016 mm in diameter and designated, in this paper, as "1" and "2" for the top and bottom plate, respectively. In operation, the cold plates are controlled manually by circulating coolant from individual constant-temperature baths. Surrounding the plates is a secondary guard consisting of an environmental chamber not shown in Fig. 2 (see Fig. 1).

As illustrated in Fig. 2, there are three independent circular heaters embedded at different radii in the hot-plate assembly. The circular line-heat-source for the meter plate (shown schematically at the mid-plane of the meter plate) is located at a radius of 143.4 mm and is a ribbon-type heating element, 0.1 mm thick by 4 mm wide, having an electrical resistance of approximately 58 Ω at room temperature. In the guard plate, there are two circular line-heat-sources located at radii of 262.35 and 401.1 mm and designated as "inner" and "outer" guard heaters, respectively. The guard heater elements are located in metal sheaths, 1.59 mm in diameter, and have approximate electrical resistances of 72 Ω and 108 Ω, respectively, at room temperature. The grooves for the guard heaters have been filled with a high-temperature epoxy.

The temperature of the meter plate is measured using a capsule platinum resistance thermometer (PRT) located at the mid-plane edge of the meter plate (Fig. 2). The temperature difference across the guard gap is measured using type-E (nickel–chromium, constantan) PTFE[1]-insulated thermocouples connected differentially in a 4-junction thermopile. The alternating pairs are located at radii of 201.1 mm and 205.45 mm, respectively. Likewise, the temperature difference for the outer guard heater is determined by a 4-pair type E thermopile; alternating pairs are located at radii of 268.7 mm and 394.75 mm, respectively (Fig. 2). Additional details for the arrangement of the temperature sensors and heaters have been discussed previously [3,4].

For the two-sided mode of operation, two test specimens – ideally, with the same nominal density, size, and thickness – are placed between the hot and cold plates (Fig. 2). The apparatus, with proper guard control, provides essentially one-dimensional heat flow and, with the two cold plates at the same temperature, equal heat flow ($Q/2$) though each specimen. With steady-state temperatures and heat flow, the thermal conductivity ($\lambda$) or unit resistance ($R$-value, $L/\lambda$) is then determined by Fourier's equation, which is

[1] Polytetrafluoroethylene.