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# Tuning PID controllers for higher-order oscillatory systems with improved performance

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ABSTRACT

In this paper, model based design of PID controllers is proposed for higher-order oscillatory systems. The proposed method has no limitations regarding systems order, time delays and oscillatory behavior. The reduced model is achieved based on third-order modeling and selection of coefficients through the use of frequency responses. The tuning of the PID parameters are obtained from a reduced third-order model; the procedure seems to be simple and effective, and improved performance of the overall system can be achieved. Three simulation examples and one real-time experiment are included to demonstrate the effectiveness and applicability of the proposed method to systems with oscillatory behavior.


## 1. Introduction

Many industrial processes are of high order and need to be converted into lower order to design effective controllers. Existing model order reduction techniques for lower-order controller design such as Proportional–Integral–Derivative (PID) have certain limitations in terms of their applicability, reliability and accuracy. The model reduction technique should be suitable and competent, and able to provide most of the dynamics of higher-order systems. Most commonly used controllers in the process control industry are PID. The main reason for PID being used is its remarkable effectiveness, relatively explicable structure and simplicity of implementation in practice by process and control engineers. According to [1], more than 95% of controllers in the different industries are PI or PID up to the last decade. However, there is not an easy way to find the optimum parameters of a PID controller, and a straightforward procedure is needed. Consequently, the best methods for tuning PI/PID controllers for higher-order systems are always adapted from time to time as a result of research algorithms [2,3]. An effective and considerable amount of research work has been reported in the past on the tuning of PID controllers. Some of them are Ziegler–Nichols step response, Ziegler–Nichols ultimate cycling, Cohen–Coon, internal model control, and error-integral criteria. However, these tuning methods use only a small amount of information about the systems, and often do not provide good tuning for higher-order systems. Tuning of PID controllers based on gain and phase margin (GPM) specifications is not a new approach and it has been reported in the literature (e.g. see [4–7]). In GPM the solutions are normally obtained numerically or graphically by trial and error. Such methods are certainly not suitable for systems having infinite phase crossover frequencies (e.g. systems without time delays and having number of zeros less than the number of poles by one). The disadvantage of the GPM method is that the transfer function of the controlled systems is restricted to the first (second) order plus time delay. However, the gain margins and phase margins may generate very different performances for different kinds of process dynamics [1]. Such types of the problem have not been addressed in the literature and no general procedure can be illustrated on how to set the values of gain and phase margins to have the desired closed-loop performance for systems having complex dynamics.

In control system design, the reduced model makes the synthesis, analysis and design of a controller simpler. That is why the reduction of high-order systems to reduced-order systems has been a topic of interest of many researchers (e.g. [8–11]). Simple PI/PID controllers are normally to be preferred for high-order linear time-invariant systems. Therefore, there is a desire to have methods available for the design of controllers which have order significantly less than the order of the plant they are controlling.

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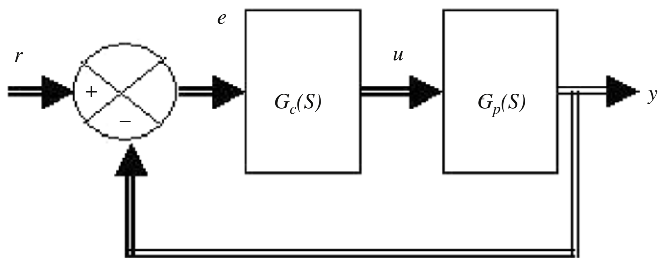


**Fig. 1.** Structure of systems with a controller.

The contributions of this paper are two-fold. First, the reduced-order model is achieved for linear time-invariant systems. The presented model order reduction technique is based on frequencies that lie in the transient region of the systems. The methods not only preserve the stability and steady state response of the system in the reduced-order model but also perform better than the available methods in the literature. Second, a PID controller is designed from the reduced-order model for the original higher-order system; it guarantees the stability of the resulting closed-loop system. Thus, the problem of stabilization of the original system when the controller is designed from the reduced-order model is addressed. The proposed method is demonstrated on three high-order oscillatory examples, including those that cannot be readily solved with the existing tools. A real-time experiment is carried out on level control system and the performance of the overall system with the designed controller is checked.

The organization of this paper is as follows. Section 2 illustrates the method of model order reduction based on the original system's frequency response. The tuning of the parameters of the PID controller are addressed in Section 3 while Section 4 includes simulation examples. Section 5 is devoted to real-time implementation, and the general conclusions are summarized in Section 6.

## 2. Model order reduction

Suppose the transfer function of the high-order oscillatory system $G_P(s)$ is available and the objective is to design a single-loop PID controller. The structure of the single-loop controller configuration is as shown in Fig. 1. It is common practice to reduce the high-order model into a first- or second-order model with time delay. In many cases, such practices lack the capture of the all transient information of the systems, or the reduced-order model's steady state gain is simply not equal to the steady state gain of the original high-order system. Even if the higher-order model is reduced into lower order, a controller designed based on the reduced model may require a trial and error procedure. In fact, the first-order models are widely used for low-order modeling and they carry only real poles. Hence they are unable to generate peaks in the frequency response of oscillatory processes.

Let the higher-order model with open-loop transfer function $G_P(s)$ be of the following structure:

$$G_P(s) = \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_m}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n} \mathrm{e}^{-t_d s}, \tag{1}$$

or with $m = 0$

$$G_P(s) = \frac{b_0}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n} \mathrm{e}^{-t_d s} \tag{2}$$

where, for time-invariant systems, the $a_i$'s and $b_i$'s are constants, $m$ and $n$ are integers with $m \leq n$, and $t_d$ is time delay. In this paper, the system given by Eq. (1) is reduced into a third-order transfer function $G_{PR}(s)$ as

$$G_{PR}(s) = \frac{\beta_0 s^2 + \beta_1 s + \beta_2}{\alpha_0 s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3} \mathrm{e}^{-t_{dr} s}, \tag{3}$$

and model given by Eq. (2) as

$$G_{PR}(s) = \frac{1}{\alpha_0 s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3} \mathrm{e}^{-t_{dr} s}, \tag{4}$$

where, for time-invariant systems, the $\beta_i$'s and $\alpha_i$'s are constants, and $t_{dr}$ is the time delay of the reduced model. The controller design procedure for the reduced model given by Eq. (4) is considered here and the design procedure of reduced model with one or more zeros like Eq. (3) is presented in the Appendix. In Eq. (4), the four unknowns ($\alpha_0, \alpha_1, \alpha_2$ and $\alpha_3$) need to be determined. The system given by Eq. (2) in the frequency domain can be written as

$$G_P(j\omega) = \frac{b_0}{(j\omega)^n + a_1 (j\omega)^{n-1} + a_2 (j\omega)^{n-2} + \cdots + a_n} \mathrm{e}^{-jt_d\omega}. \tag{5}$$

The magnitude and phase of the system given by Eq. (5) are

$$|G_P(j\omega)| = \frac{b_0}{\sqrt{A^2 + B^2}}, \tag{6}$$

and

$$\angle G_P(j\omega) = -\arctan\left(\frac{B}{A}\right) - t_d\omega, \tag{7}$$

respectively. The terms $A$ and $B$ are $A = a_n - a_{n-2}\omega^2 + a_{n-4}\omega^4 - a_{n-6}\omega^6 + \cdots$ and $B = a_{n-1}\omega - a_{n-3}\omega^3 + a_{n-5}\omega^5 - \cdots$. The magnitude and phase for the reduced-order model given by Eq. (4) are

$$|G_{PR}(j\omega)| = \frac{1}{\sqrt{A_r^2 + B_r^2}}, \tag{8}$$

and

$$\angle G_{PR}(j\omega) = -\arctan\left(\frac{B_r}{A_r}\right) - t_{dr}\omega, \tag{9}$$

where $A_r = \alpha_3 - \alpha_1\omega^2$ and $B_r = \alpha_2\omega - \alpha_0\omega^3$. The reduced model given by Eq. (8), can be written as

$$\begin{pmatrix} \omega^6 & \omega^4 & \omega^2 & \omega^0 \end{pmatrix} \begin{pmatrix} \alpha_0^2 \\ \alpha_1^2 - 2\alpha_2\alpha_0 \\ \alpha_2^2 - 2\alpha_1\alpha_3 \\ \alpha_3^2 \end{pmatrix} = \frac{1}{|G_{PR}(j\omega)|^2}. \tag{10}$$

The four frequencies of the original model given by Eq. (5) on transient response including bandwidth and phase crossover are used to find the values of $\alpha_0$ to $\alpha_3$. These four frequencies are $\omega_1$, $\omega_2$, $\omega_3$ and $\omega_4$ at phase $-\pi/4, -\pi/2, -3\pi/4$ and $-\pi$, respectively, such that $|G_P(j\omega)| \cong |G_{PR}(j\omega)|$. These frequencies are used to find the corresponding gains of the system and hence phase due to delay time is not considered. The least square approach is used to fit the reduced-order model with the original model.

Define $\Theta$ and $\Phi(\omega)$ as a vector of unknowns and matrix of frequencies, respectively, as

$$\Theta = \begin{bmatrix} \alpha_0^2 & \alpha_1^2 - 2\alpha_2\alpha_0 & \alpha_2^2 - 2\alpha_1\alpha_3 & \alpha_3^2 \end{bmatrix}^{\mathrm{T}} \tag{11}$$

$$\Phi(\omega) = \begin{bmatrix} \omega^6 & \omega^4 & \omega^2 & \omega^0 \end{bmatrix}^{\mathrm{T}}, \tag{12}$$

where T represents the transpose. By least squares, the unknowns are estimated as

$$\Theta = \left[\sum_{\omega=\omega_1}^{\omega_N} \Phi(\omega)\Phi^{\mathrm{T}}(\omega)\right]^{-1} \left[\frac{\sum_{\omega=\omega_1}^{\omega_N} \Phi(\omega)}{|G_P(j\omega)|^2}\right]. \tag{13}$$