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Sub-optimal PID controller settings for FOPDT systems with long dead time

Jan Cvejn *

University of Pardubice, Faculty of Electrical Engineering and Informatics, Studentská 95, 532 10 Pardubice, Czech Republic

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ABSTRACT

We propose a method for setting up PI and PID controllers based on stable FOPDT process model, where dead-time dynamics is manipulated without approximation. The main idea used is a partial compensation of the system dynamics, which makes possible obtaining simple tuning rules. Remaining unknown controller parameters are determined on the basis of the modulus optimum and the minimum ISE criterions. Besides the performance indices, quality of the settings is also evaluated by the stability margin. Although optimal values of the parameters are valid for the reference tracking problem, a compensation of the disturbance lag that preserves the stability margin is proposed for the disturbance rejection problem.

1. Introduction

Dynamics of many industrial processes can be well approximated by the stable first-order plus dead time (FOPDT) transfer function:

$$S(s) = \frac{K}{Ts+1}e^{-\tau s}, \tag{1}$$

where K is the system gain, T is the time constant and τ is the dead time parameter. The model allows simple experimental identification from the step response, which can be in most cases easily measured. Simple methods based on coincidence in one or more points and more complex methods suitable for noisy data are described in the literature [1,5].

For tuning PID controllers based on the model (1) many approaches exist, see e.g. [1] for description of the most important methods. A comprehensive survey of known formulas is available in [10]. Early methods were derived from empirical requirements on the step response, such as one-quarter decay ratio [2,9], step-response overshoot [11] or from integral criterions in time domain with approximation of the dead-time dynamics [6,13]. These methods, however, usually work well only for a limited range of the ratio τ/T.

Among recent methods for the model of type (1) without approximation of the delay term the design with given gain and phase margins [4] and LQR design [5] should be mentioned. Alternative ways for systems with long time delay include internal model control [7], Smith predictor and λ-tuning [1]. These approaches, however, require implementation of delay in the control system.

The method for setting up PI controller parameters based on cancellation of the factor $(Ts+1)$ was proposed by Haalman [3]. In this method the dead-time dynamics is manipulated without approximation. Good reference tracking performance is achieved, but on the other hand, pure results may be observed for rejection of load disturbances, see [1]. In a similar way it is possible to compensate dynamics of the second order plus dead-time system by a PID controller.

In this paper, we utilize Haalman's idea of pole compensation for designing optimal PI and PID controller parameters for the model (1). The pole compensation fixes the value of one parameter of the controller. We adjust the remaining controller parameters to meet analytic design criteria. We show that in this case especially the modulus optimum criterion leads to a simple choice of the parameters and to a control loop with very good practical properties. In this case, derivative term of the controller increases both the performance and the stability margin. Besides the settings for optimal reference tracking we propose a suitable compensation for efficient rejecting load disturbances.

We work with the serial PID controller with the transfer function

$$R(s) = K_C\left(1+\frac{1}{T_I s}\right)(1+T_D s). \tag{2}$$

Transfer function of general (parallel) PID controller has the form

$$R(s) = \widetilde{K}_C\left(1+\frac{1}{\widetilde{T}_I s}+\widetilde{T}_D s\right). \tag{3}$$

Transformation between the parameters from serial to parallel type is easily obtained as

* Tel.: +420 466 037 104; fax: +420 466 036 241.
E-mail address: jan.cvejn@upce.cz.

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$$\widetilde{K}_C = \alpha K_C, \quad \widetilde{T}_I = \alpha T_I, \quad \widetilde{T}_D = \frac{1}{\alpha} T_D, \tag{4}$$

where

$$\alpha = \left(1 + \frac{T_D}{T_I}\right). \tag{5}$$

The inverse transform, however, does not always exist [1].

2. Objectives of the design

If the pole $-1/T$ of the system (1) is compensated, it is possible to adjust remaining parameters of the controller to meet design criteria. Due to partial compensation of the dynamics the space of optimized controller parameters is reduced, so the settings proposed can be only sub-optimal. Also, we use the serial structure of PID controller, which poses some constraints on the parameters variability. On the other hand, this structure makes possible obtaining simple tuning rules, which is very desirable with respect to simplicity of the model (1) and practical usability.

We are using two design specifications, which are both often mentioned in the PID control literature: (1) the modulus optimum criterion and (2) the minimum integral square error. Analytical controller design procedures based on these criterions are well known if the system dynamics is without dead time. Otherwise, it is possible to replace the dead-time term using truncated Taylor or Padè expansion and apply the standard procedure. In general, for long dead time the order of the approximation plant then has to be high and the design procedure may become impractical. However, it is true that the number of terms of the expansion needed at application of the first criterion corresponds to the number of unknown parameters of the controller and not to the time delay [18]. Alternatively, optimal values of the controller parameters for given plant can be found numerically.

The modulus optimum criterion (MO) introduced in [16] requires that the amplitude of the closed-loop frequency response is close to one for low frequencies. If the closed-loop frequency response is decreasing, this condition is analogous to the requirement that the frequencies in the reference input are passed in the broadest possible range. Such a closed-loop system is able to respond quickly to changes of the reference input.

We define the order $n_0(f(x))$ of a function $f(x) \in C_\infty$ in the origin as the zero-based index of the first nonzero coefficient in the Taylor expansion of $f(x)$ in the origin. Let us write the closed-loop frequency response in the form

$$T(\omega) = \frac{L(i\omega)}{1 + L(i\omega)} = \frac{1}{1 + 1/L(i\omega)}, \tag{6}$$

where $L(s)$ is the open-loop transfer function. If $L(s)$ contains a pole in the origin, which is necessary to achieve asymptotically zero regulation error, for $\omega \to 0$ holds $L(i\omega) \to \infty$ and thus $\lim_{\omega\to 0} |T(\omega)|^2 = 1$. Therefore, it is possible to write $|T(\omega)|^2$ as

$$|T(\omega)|^2 = 1 + H(\omega), \tag{7}$$

where $H(\omega) \in C_\infty$. Maximal flatness of the closed-loop frequency-response modulus is then equivalent to the requirement that

$$n_0(H(\omega)) \to \max. \tag{8}$$

As the second criterion we are using the minimum integral square error (ISE) of the response to a step change of the reference signal:

$$J = \int_0^\infty e^2(t)dt \to \min, \tag{9}$$

where $e(t) = w(t) - y(t)$ is the regulation error. Integral of the square regulation error is usually considered as a good measure of quality of the controller settings, although the criterion leads to rather oscillating responses. Therefore, alternative integral criterions were proposed, such as IAE:

$$J = \int_0^\infty |e(t)|dt \to \min. \tag{10}$$

For the plant (1) several approximate ISE and IAE-optimal settings were published, e.g. in [6,13]. However, approximate settings are valid only for limited ratio τ/T.

Besides performance objectives, the design has to respect stability requirements. As the stability margin we consider the distance of the open-loop Nyquist plot from the critical point $[-1, 0]$, i.e. the value

$$\gamma = \inf_{\omega\in(0,\infty)} \{1 + L(\xi)\}, \quad \gamma \in [0, 1]. \tag{11}$$

The reciprocal value of γ is known as the sensitivity. In general case it is recommended that the sensitivity is in the range from 1.3 to 2 [1].

2.1. Optimal PI controller design

Consider the reference tracking control problem in Fig. 1. If we compensate the factor $(Ts + 1)$ by a PI controller with transfer function of the form

$$R(s) = \kappa \frac{T}{\tau K}\left(1 + \frac{1}{Ts}\right) \tag{12}$$

the open-loop transfer function is

$$L(s) = \frac{\kappa}{\tau s} e^{-\tau s}, \tag{13}$$

where κ is a tuning parameter. The corresponding frequency response can be written as

$$L(\xi) = \frac{\kappa}{i\xi} e^{-i\xi} = \frac{\kappa}{\xi} e^{-i(\xi+\pi/2)} = -\kappa\left(\frac{\sin\xi}{\xi} + i\frac{\cos\xi}{\xi}\right), \tag{14}$$

where $\xi = \tau\omega$ is normalized frequency. The ultimate normalized frequency, where $\arg L(\xi) = -\pi$, is $\xi_c = \pi/2$. From (14) easily follows that the gain and the phase margins are

$$\alpha = \frac{\pi}{2\kappa}, \quad \beta = \frac{\pi}{2} - \kappa \tag{15}$$

and holds

$$\frac{1}{\alpha} + \frac{2}{\pi}\beta = 1. \tag{16}$$

The corresponding Nyquist plot is dependent only on a single parameter κ, which can be adjusted so that sufficient stability margin is guaranteed and performance objectives are fulfilled. Since the geometrical shape of the open-loop Nyquist is κ-invariant, it is easy in this configuration to inspect closed-loop stability and robustness.

It is also possible to choose directly $\alpha \in (1, \infty)$ as a tuning parameter. Figs. 2 and 3 show the open-loop frequency responses and corresponding step responses for α = 2, 3 and 4, $\tau = 80$ s. Obviously, the settings for $\alpha \in [2, 4]$ conform to practical requirements in most cases.

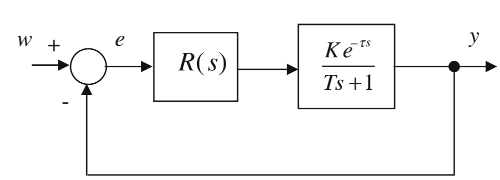

Fig. 1. Control scheme for reference tracking.