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A PID controller set of guaranteeing stability and gain and phase margins for time-delay systems

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ABSTRACT

The stabilizing parameter sets and the guaranteed gain-margin (GM) and phase-margin (PM) regions of proportional-integral-derivative (PID) controllers for a class of processes with time-delay are discussed in the paper. The admissible range of stabilizing proportional-gain is first derived by a version of Hermite–Biehler Theorem and the evaluation of some properties of the functions involved in the closed-loop characteristic equation. Then, the stabilizing region in integral-derivative plane, for a fixed proportional-gain, is drawn and identified directly in terms of a graphical stability criterion applicable to time-delay systems. Further, in the stabilizing region, the gain-margin and phase-margin specifications are considered using the same strategy as drawing the stability boundary lines, based on the technique of gain-phase margin tester (GPMT). Illustrating examples are followed in each design step to show the effectiveness of the method.

1. Introduction

PID controllers, which pertain to the classical control theory, are widely applied in industrial process control field, although the modern and post-modern control theories have emerged for a few decades. This is due to the fact that PID controller is simple in structure and it is possible to achieve a satisfactory control effect and acceptable robustness [1]. On the other hand, time-delay is a common phenomenon in many processes. The existence of time-delay in a control loop is a source of instability and performance deterioration [2], because it leads to a characteristic quasi-polynomial of the closed-loop with infinite number of roots [3].

The tuning methods of PID controllers have been studied intensively in the past, such as the well-known Ziegler–Nichols tuning rule [4] for the first order plus time-delay transfer functions, which models a wide class of processes possessing an S-shape reaction curve in step responses. Another research line for the design of PID controllers is to determine the stabilizing parameter set of PID controller, this set was first shown in [5] as convex polygons for delay-free systems by an extension of the Hermite–Biehler theorem presented by Pontryagin [6]. Then, the approach was applied to first-order plus time-delay systems and the convex polygon property was extended to this case [7]. By using the Nyquist stability criterion, the same results as those in [7] were obtained which gave an alternative simple derivation [8]. The technique employed in [7] was also generalized to the second-order-integrating processes with time-delay [9]. For the second-order time-delay plants [10] and the nth-order all pole time-delay plants [11], a graphical approach was applied to draw the stabilizing regions of PID controllers, exhibiting a simple characterization in determining the stabilizing parameters of PID controllers. The determination of stabilizing regions for second-order time-delay plants with one zero was considered in [12] in both process and controller parameter plane via the evaluation of some properties of the functions relating to the plant. In [13], a complete characterization of stabilizing regions of a general second-order quasi-polynomial with a single delay was derived using the root location approach. The authors in [14] expanded the Hermite–Biehler method to arbitrary linear delay systems, and the stabilizing regions were obtained by searching the set of controller parameters. Since then, some research works have focused on the stabilization of arbitrary delay systems using PID controllers [15–18]. The D-decomposition approach was extended to this case in a compact way in [15], and the entire set of stabilizing PID controller parameters was computed for both retarded and neutral systems. In [16,17], by employing an extended Hermite–Biehler Theorem, the results on the design of PID (P, PI, PID separately in [17]) controllers were obtained via a procedure of linear programming, and the computational characterizations are analogous to the Youla parameterization of all stabilizing controllers for rational plants [17]. Based on the inverse Nyquist plot, the computation of stabilizing PID gain regions was carried out in [18].

The gain and phase margins design of control systems is an old topic that was first introduced in [19] for SISO systems. In [20], PID controller designed to satisfy gain and phase margins was presented. Recent achievements on the issue can be found in [21–26]

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and references therein, aiming at giving new or improved design procedure.

In the present work, we first consider a class of time-delay plants with zeros to determine the stabilizing parameter set of PID controllers. In author's previous works [10,11], the zeros in the transfer function of the plant were not included for the simplicity of derivation. The necessary conditions are described explicitly for the proportional-gain interval potentially providing a stable region in the plane of integral and derivative gains using a version of Hermite–Biehler Theorem. Based on a graphical stability criterion applicable to quasi-polynomial, the stabilizing region in integral-derivative plane is drawn and identified for a fixed proportional-gain in the stabilizing interval. In addition, we further discuss, in the stabilizing region, the classical performances of the gain-margin and phase-margin by the same strategy as stability analysis. The graphical tuning presented in this paper is direct and flexible in the selection of PID parameters.

The rest of the paper is organized as follows. In Section 2, the problem statement and some properties of the plant are provided. Then, Section 3 proves the explicit stabilizing interval of proportional-gain, and Section 4 discusses the stabilizing region in the plane of integral and derivative gains. In Section 5, the traditional performances of gain-margin and phase-margin are considered in the stabilizing region. Finally, concluding remarks are given in Section 6.

2. Problem formulation and preliminaries

Consider the general structure of SISO LTI unity feedback with a so-called GPMT [27] as shown in Fig. 1, where $G(s)$ and $C(s)$ are the transfer functions of the process and controller, respectively, which are given by

$$G(s)=\frac{k\prod_{j=1}^{m}(\tau_j s+1)}{s^{\nu}\prod_{i=1}^{n}(T_i s+1)}e^{-Ls},\quad \nu=0,1,2 \tag{1}$$

$$C(s)=\frac{K_i+K_p s+K_d s^2}{s} \tag{2}$$

where $k>0$ is the gain of the process, τ_j, T_i the time constants, L the time-delay, m, $n+\nu$ the orders of the numerator and denominator of $G(s)$, ν the number of the integrators, representing the type of the process, K_i, K_p and K_d stand for the integral, proportional and derivative, respectively, gains of PID controller, and $Ae^{-j\varphi}$ represents the GPMT, which provides information on plotting the boundary lines of constant gain-margin and phase-margin in the parameter plane of PID controller, corresponding to the following three cases: (a) Setting $A=1$, one obtains the boundary lines for a given phase-margin φ. (b) Setting $\varphi=0$, the boundary lines for a desired gain-margin A. And (c) To find the stabilizing boundary lines, one needs to set $A=1$ and $\varphi=0$ simultaneously. In practical control systems, the block of GPMT does not exist, it is only used for the design of PID controllers with desired gain and phase margins.

The problem of interest in this paper is to determine the three gain parameters of PID controller given in (2) such that the closed-loop system depicted in Fig. 1 with $A=1$ and $\varphi=0$ is stable. Furthermore, with the stabilizing ranges of the gains of PID controller being known, one tunes the parameters of PID controller to satisfy the gain-margin or phase-margin performance by setting $\varphi=0$ or $A=1$, respectively.

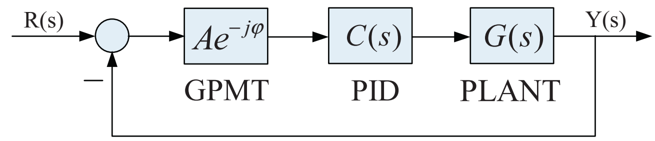

Fig. 1. General structure of unity feedback.

In the following, the stability problem of the closed-loop in Fig. 1 for PID controlled time-delay system is first considered. The closed-loop characteristic quasi-polynomial is given by

$$D_1(s)=s^{\nu+1}\prod_{i=1}^{n}(T_i s+1)+k(K_i+K_p s+K_d s^2)\prod_{j=1}^{m}(\tau_j s+1)e^{-Ls}$$

Multiplying $D_1(s)$ by e^{Ls} yields

$$D(s)=s^{\nu+1}\prod_{i=1}^{n}(T_i s+1)e^{Ls}+k(K_i+K_p s+K_d s^2)\prod_{j=1}^{m}(\tau_j s+1) \tag{3}$$

The following two conditions are necessary for the stability of $D(s)$ in (3).

Condition 1. A minimal requirement for any control design is that the delay-free closed-loop system be stable.

Condition 2. It is necessary that $D(s)$ have no more than a bounded set of zeros in the open right-half plane for stability. This holds if $D(s)$ has a principal term $a_{pq}s^p e^{qs}$, where $p=n+\nu+1$ and $q=1$ in (3), it exists if $m\le n+\nu-1$, and the coefficient function $\chi_p(s)$ of s^p, where $\chi_p(s)=e^{Ls}\prod_{i=1}^{n}T_i$, for $m<n+\nu-1$ and $\chi_p(s)=e^{Ls}\prod_{i=1}^{n}T_i+kK_d\prod_{j=1}^{m}\tau_j$, for $m=n+\nu-1$, has all the zeros in the open left-half plane. This happens if one of the following assumptions is satisfied

(A) $m<n+\nu-1$.
(B) $m=n+\nu-1$ and $\left|kK_d\prod_{j=1}^{m}\tau_j/\prod_{i=1}^{n}T_i\right|<1$.

Note that Condition 2 is presented by Pontryagin in [6]. The quasi-polynomial (3) under Assumption (A) is a quasi-polynomial of retarded type and under Assumption (B) belongs to a class of quasi-polynomial of neutral type [16].

Now, let us evaluate some expressions involved in (3) that will be used in the following sections.

Substituting $s=j\omega$ into (3) yields

$$D(j\omega)=(j\omega)^{\nu+1}\prod_{i=1}^{n}(jT_i\omega+1)e^{jL\omega}+k(K_i+jK_p\omega-K_d\omega^2)\prod_{j=1}^{m}(j\tau_j\omega+1) \tag{4}$$

Multiply both sides of (4) by $\prod_{j=1}^{m}(-j\tau_j\omega+1)$ and let $z=L\omega$, one gets

$$\begin{aligned}D^*(jz)&=(jz/L)^{\nu+1}\prod_{i=1}^{n}(jT_i z/L+1)\prod_{j=1}^{m}(-j\tau_j z/L+1)e^{jz}+k(K_i+jK_p z/L-K_d z^2/L^2)\prod_{j=1}^{m}(\tau_j^2 z^2/L^2+1)\\&=(jz/L)^{\nu+1}(A(z)+jB(z))(C(z)+jD(z))e^{jz}+k(K_i+jK_p z/L-K_d z^2/L^2)E(z)\\&=(jz/L)^{\nu+1}[M(z)\cos z-N(z)\sin z+j(M(z)\sin z+N(z)\cos z)]+k(K_i+jK_p z/L-K_d z^2/L^2)E(z)\end{aligned} \tag{5}$$