Project Scheduling Approach to Production Planning

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Abstract
We suggest a novel approach to modelling and solving production planning (PP) problems in make-to-order production environments. This approach unifies the capacity and the material flow oriented aspects of PP. Orders are modelled as projects that compete for limited resources. The projects consist of networks of variable-intensity activities which may require several resources at the same time. The goal is to generate production plans that satisfy all the temporal and resource constraints and minimize additional or external resource usage over the planning horizon. The model is applicable at different aggregation levels of PP, as we show with two real-life case studies. Although the capacity and material flow aspects of PP are coupled, a special solver developed for the project model efficiently solves problems of real-life sizes.

Keywords:
Production, planning, project

1 INTRODUCTION

Production planning (PP) matches future production load and capacities by generating plans that determine the flow of materials and the use of resources over a given planning horizon.

Finding an appropriate match between load and capacities is a chicken-and-egg problem. On the one hand, the actual load – and the corresponding flow of materials – should fit to the available resource capacities. On the other hand, the level of capacities should be determined so as to meet the demand over time. The problem is usually settled by fixing either the load or the capacities [1].

However, this approach can hardly be taken nowadays when the traditional boundaries of companies are getting dissolved. In the era of supply chain management, virtual enterprises and production networks the tasks of PP may cross the organizational boundaries of the firms. Decisions on the use of resources should concern both internal and *external* capacities; the internal flow of materials should be synchronized with the incoming and outgoing flows [2]. All this makes the PP problem extremely hard to solve. Conversely, the complex situations call for efficient, robust decision support methods at each node of a production network [2,3]. Hence, there is a need of intuitive and flexible models *and* fast, reliable solution techniques that scale-up well also to large problem instances. An important practical requirement for any new method is that it should be able to work by using data stored in existing – so-called legacy – production information systems.

There are long-standing recipes to handle complexity. *Aggregation* removes details in the representation of products and orders, production processes, resource capacities, and time [1,4]. Similar problems formulated with more details are limited by shorter planning horizon. The solution is generated in a process where higher level solutions provide constraints to lower level problems. *Decomposition* separates PP problems into a resource and a load oriented subproblem. Decomposition is usually applied on several levels of aggregation. E.g., decomposed planning functions are traditionally termed as master production scheduling and material requirements planning on the side of the load, whereas rough-cut capacity planning and capacity requirements planning on the side of resources [1].

Below we suggest a novel approach to modelling and solving long- and medium-term capacity and production planning problems. Our method is based on a generalized version of the *resource-constrained project scheduling* problem [5] and unifies the resource and the material flow oriented aspects of PP. By introducing *activities*, we handle together resource and temporal constraints. Hence, planning can work without using lead time estimates that, in a turbulent environment, cannot represent individual orders any more [6]. The project-based approach captures the strong goal-oriented nature of *make-to-order* and *engineering-to-order* production.

The paper is structured in the following way: In Section 2 the project model is presented along with the main principles of the solution technique. Then we describe how to solve PP problems in two different production environments by taking the suggested project-based approach (Sections 3 and 4). Finally, conclusions of the two case studies are drawn in Section 5.

2 THE BASIC MODEL

2.1 Resource-constrained project scheduling

Resource-constrained project scheduling problems are concerned with scheduling a number of discrete activities, each requiring some resources. Constraints due to the limited capacities of resources and precedence relations between the activities are prescribed. The classical model assumes fixed activity durations and a constant rate of resource usage during the entire processing of every activity [5,7]. However, in *aggregate* planning the above assumptions cannot be taken and there is also no need to generate detailed solutions for future periods that will certainly be different to what is anticipated. Hence, after studying real PP problems we extended the classical model by allowing (1) *preemption* of activity execution, (2) *variable-intensity* activities, and (3) *continuously* divisible resources.

2.2 Activities and resources

An instance of the problem is given by a set $N = \{1,\ldots,n\}$ of *activities*, a set $R = \{1,\ldots,r\}$ of continuously divisible and renewable *resources*, and a directed acyclic graph $D = (N, A)$ representing *precedence constraints* among the activities. Each activity $i \in N$ must entirely be processed within its *time window*: between its earliest starting time e^i and deadline d^i.

Each activity may require the simultaneous use of some *resources*. The entire processing of activity i requires a total of r_k^i units of resource k, for each $k \in R$. The *intensity* of each activity may vary over time, and the resource usage is proportional to the intensity. If x_t^i is the intensity of activity i in time period t, then it requires $r_k^i * x_t^i$ units of resource k in that period. However, the intensity of executing an activity is limited: in any time period $t \in [e^i, d^i]$ at most $a^i \leq 1$ fraction of activity i may be completed.

The *capacity* of each resource $k \in R$ is fixed period by period over the horizon. In each time period t, a certain internal capacity of each resource k is available. Internal resource capacities can be used free of charge. Additional external capacities are also available, but at the expense of some cost per resource units.

2.3 Generalized precedence relations

The most typical precedence constraint between a pair of activities i, j prescribes that activity i must finish before activity j may start. However, in practical problems a precedence relation between a pair of activities may have the following form: start activity j only if 25% of activity i has been completed, or the last 30% of activity j can be done only after 60% of activity i was completed.

Our model supports the above kind of generalized precedence relations. Note that there could be several relations between a pair of activities i, j.

2.4 Optimization criteria

In our basic model the *cost of using external resources* is to be minimized. This optimization criterion is motivated by practical applications that will be described in Sections 3 and 4.

We note here that classical optimization criteria, like project *duration*, *maximum tardiness* or *weighted tardiness* fit also in the proposed framework.

2.5 Problem statement

The problem consists of determining for each activity i an intensity x_t^i in each time period $t \in [e^i, d^i]$ such that $0 \leq x_t^i \leq a^i$, $\sum x_t^i = 1$, all the precedence constraints among the activities are fulfilled, the resource demands do not exceed the resource availabilities in any time period, and the total cost of using external capacity is minimized.

For a simple example, see Fig. 1 with data of two projects. Activity 1 and 2 are linked by a precedence relation. The resources have a unit capacity.

	e^i	d^i	a^i	r_1^i	r_2^i
i=1	1	3	1/2	1	0
i=2	1	3	1	0	1
i=3	1	3	1/2	2	0

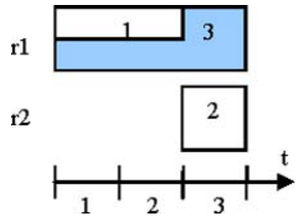

Figure 1: Data of two sample projects and the solution.

The above problem has been formalized as a mixed integer-linear program. A detailed analysis of this mathematical program has shown that the problem is NP-complete in the strong sense. However, the analysis resulted also in a linear programming re-formulation with some cutting planes. The solution method uses them in a branch-and-cut algorithm that finds optimal solutions. Our extensive numerical tests confirmed that branch-and-cut is a viable approach for solving even very large problem instances. For details of the analysis and solution method, see [8].

3 APPLICATION TO PRODUCTION AND CAPACITY PLANNING

3.1 Background

In Case Study 1, we consider a factory with a long tradition in manufacturing special, one-of-a-kind equipment for producing a mass product for household use. The factory handles accepted orders as *projects* that must be completed between their release dates and deadlines.

The *internal* resources are well-organized and stable. At subcontracting partners, there are *external* capacities for all resources, but for a given unit costs. The crucial problem is to determine the timing and resource assignments of the activities of all the projects so as to satisfy the temporal and resource constraints, and to minimize the cost of external resource usage. Production and capacity planning should be supported in an integrated way, at two levels of aggregation:

- On the long term, with a 1-1.5 year horizon, by considering the various departments (like mechanical design, components machining, mechanical assembly, electric design, electric assembly, installation, etc.) as resources.
- On the medium term, with a quarter horizon, by considering the groups of machine and labour resources of components machining.

The planning problems should be solved on both levels with a rolling horizon, since the actual states of the projects are reported week by week, and the planners have to make new plans that take into account the deviations and new requests as well.

3.2 The planning problem

In terms of our model presented above, each resource has an inside capacity (given in work-hours per week) in each week of the horizon. Inside capacities can be used free of charge. Further on, additional capacities of subcontractors can be used at some extra cost. However, these parameters may vary from week to week. The actual values of capacities and prices are influenced by several factors such as maintenance, holidays, urgency, market position etc.

Each project consists of several *activities*. Each activity may require a number of different resources. Each