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SCIENCE DIRECT
Computers and Chemical Engineering 27 (2003) 1693–1706
Computers & Chemical Engineering
www.elsevier.com/locate/compchemeng

Stochastic models and numerical solutions for production planning with applications to the paper industry

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Received 16 April 2003; accepted 16 May 2003

Abstract

This work is concerned with models and numerical algorithms for production planning of systems under uncertainties. Using stochastic processes to describe the system dynamics, we model the random demand and capacity processes by two finite-state continuous-time Markov chains. We seek the optimal production rate by minimizing an expected cost of the system. Discretizing the Hamilton–Jacobi–Bellman (HJB) equations satisfied by the value functions and using an approximation procedure yield the optimal solution, which allows us to make production decisions sequentially throughout the process lifespan. Three case studies are presented. Using demand data collected from a large paper manufacturer, the optimal production policies of the paper machine are obtained for different machine capacity and demand processes.

Keywords: Production planning; Markov chain; HJB equation; Optimal policy; Numerical method; Paper manufacturing

1. Introduction

Production planning has attracted growing attention due to its increasing importance in today's highly competitive environment (see Applequist, Samikoglu, Pekny & Reklaitis, 1997; Balasubramanian & Grossmann, 2002; Bassett, Pekny & Reklaitis, 1997; Gupta, Maranas & McDonald, 2000; Petkov & Mararas, 1997; Yin & Zhang, 1997; Yin, Yin & Zhang, 1995 and the references therein). Planning requires making decisions for a manufacturing system, in which many different types of events such as operations, failures, preventive maintenance, and/or raw material supply as well as customer demand fluctuations may occur at the same time. The complexity of the system under consideration, the large amount of information needed in decision making, the possible changes in process operating conditions, the potential technological advances, and especially the randomness originated from equipment failures, demand fluctuation, and raw material variation make the task of planning very challenging. Limitations and constraints on the production procedure, capacity, and storage space, as well as the multiple options of production add more difficulties in the decision making. To better understand and more effectively deal with uncertainties from various sources require mathematical models that can characterize the unique feature of each major event.

Production planning that involves making decision is often formulated as an optimal control problem. For optimization problems under uncertainty, two most important characteristics (Bertsekas, 1976) not present in its deterministic counterpart are the need of considering risks in the model formulation and the possibility of information gathering during the decision process. To solve such a problem usually entails the minimization of the expected value of a cost function. The production system evolves as a dynamic process, and the decision obtained is a feedback control policy.

In stochastic control, the underlying system can be formulated by a stochastic dynamic system to describe

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0098-1354/03/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/S0098-1354(03)00136-4

the functional relationship between the state variables of the system and its inputs and disturbances, e.g. the surplus amount of the production system can be taken as the state variable that is affected by both production and demand rates. Many dynamic systems consist of discrete-event processes or subject to discrete-event interventions, which lead to jump discontinuity in their evolution. For instance, manufacturing systems are often associated with movements involving discontinuity influenced by such random and exogenous discrete events as demand variation and equipment failures. The behavior of the dynamic systems under different situations could be markedly different. Due to the possible large number of states of the system, and the large number of available alternatives of decisions, the problem can be very complex.

Among the various methods used to formulate such dynamics systems, Markov chains provide an especially important class of probabilistic models. Planning amounts to designing the system control policy to optimize its performance. Using Markov chains, we need to determine which one of all the possible alternative actions should be taken at each state. The action chosen affects the outcome of the random process as well as its immediate and subsequent costs. To make decision sequentially requires a policy/rule to prescribe decisions at each state of the system during the entire process lifespan. The goal is to choose the optimal policy so that an appropriate cost function can be minimized.

The importance of planning under uncertainty has been widely recognized. There is a rich literature in its theoretical development and related topics (see e.g. Gershwin, 1994; Sethi & Zhang, 1994; Yin & Zhang, 1997) and many references therein. A one-machine, one-part production system was formulated as a stochastic optimal control problem in Akella and Kumar (1986), where the demand is a constant, the state of the machine is modeled by a two-state continuous-time Markov chain, and the objective function is a discounted inventory cost over an infinite time horizon. It was shown that the optimal control was given by a single threshold inventory level. Later on, its counterpart of long-run-average cost was studied and an optimal hedging policy was obtained in Bielecki and Kumar (1988). For a discussion of hedging policies or threshold-type controls, see Gershwin (1994) and the references therein. Boukas, Yang and Zhang (1995) presented a minimax production planning model with Markov capacity process and deterministic but unknown demand rate; and obtained the optimal control under discounted cost criterion. Optimal policies for finite horizon problems can be found in Zhang and Yin (1994), in which the corresponding optimal controls were obtained using the time-dependent turnpike sets under "traceability" conditions. A combined approach of stochastic approximation with gradient estimation techniques was proposed in Yan, Yin and Lou (1994), where the gradient estimator was constructed using the method of infinitesimal perturbation analysis (Ho & Cao, 1991; see Kushner & Yin, 1997 for an up-to-date account on stochastic approximation). For large-scale manufacturing systems, the framework of hierarchical decomposition and appropriate aggregation was considered in Sethi and Zhang (1994). An in-depth study of the related singularly perturbed Markovian systems can be found in Yin and Zhang (1998). In a recent paper (Zhang, Yin & Boukas, 2001), a class of marketing-production models was proposed, which was in fact a stochastic control problem with the demand being a Poisson process. It is desirable or sometimes necessary to model the production capacity by a continuous-time Markov chain. However, the closed-form solution for such a production planning problem is often too complicated to obtain. Therefore, a numerical solution is necessary.

It has become clear that there is a great need for methods capable of handling uncertain demands, potential technology advances, random capacities, and many other random events and disturbances in the process and its environment. This is particularly true in the paper industry. Although Markov processes have been used in many operational management problems, the applications to paper industry are still scarce. This work intends to contribute in this direction. Different from the statistical quality control approach, our models delineate the evolution of the process and reflect the dynamic behavior of the underlying systems. Contingent upon the state of the system and its capacity, the decision made is affected by both earlier decisions as well as other random disturbances. We seek optimal long- and mid-term planning decisions in the manufacturing stage for dynamic systems under uncertainties and formulate it as an optimal control problem. We address issues involved in problem formulation and solution procedure; provide the associated dynamic programming equation, and present numerical approximation scheme that leads to an approximation of the optimal policy. The objective function used includes both production and holding costs and it can be easily extended to include other costs. We consider two types of uncertainty, demand and production capacity, and formulate them using finite-state Markov chains. Such an approach enables us to quantitatively describe the random and jump behavior that is common in many stochastic dynamic systems. The policy obtained allows us to make optimal decisions sequentially for each state throughout the process lifespan.

This work is motivated by the needs for better production planning in the paper industry. Using real demand data collected from a paper manufacturer, we seek mathematical models and numerical procedures applicable to the real processes. The primary tools are