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Pinch analysis for aggregate production planning in supply chains

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Abstract

Global competition has made it imperative for the process industries to manage their supply chains optimally. The complexity of the supply chain processes coupled with large computational times often makes effective supply chain management (SCM) difficult. Production system is an important component of a supply chain. This paper introduces a novel approach for aggregate planning of production in supply chains. The approach derives inspiration from pinch analysis, which has been extensively used in heat and mass exchanger network synthesis. By representing demand and supply data as composites, it gives planners greater insight into the SCM process and thus facilitates re-planning and quick decision-making. Two case studies are solved, one involving a single product and another involving multiple products on a single processor. For the first case study, optimal production plans are obtained and matched with the results obtained by solving equivalent optimization problems in GAMS®. For the second case study, an algorithm is proposed to determine the sequence of production of the multiple products. The initial guess obtained by following the algorithm reduces the computational time to one-sixth of the time otherwise taken by the solver. It may be concluded that plans obtained by pinch analysis provide either the best aggregate plans or excellent starting points to reduce the computational time for solutions by mixed integer programming formulations.

Keywords: Pinch analysis; Aggregate production planning; Mathematical programming; Supply chain management

1. Introduction

Supply chain for a business consists of all the stages involved directly or indirectly in fulfilling demand from a customer. Production planning, scheduling and distribution are some of the operations performed in a supply chain. In most cases, a planning model is developed and an equivalent optimization problem is solved using standard optimization algorithms (McDonald & Karimi, 1997). One of the difficulties faced with discrete mathematical programming is combinatorial complexity, which increases dramatically with the size of the problem.

The aim of this work is to introduce the approach of pinch analysis in aggregate planning within the overall framework for optimization of supply chains. Aggregate planning (Chopra & Meindl, 2001) aims at maximizing profit over a specified time horizon while satisfying demand.

Pinch analysis has been extensively used in chemical engineering for the optimization of various resources such as energy and water (Linnhoff & Hindmarsh, 1983; Shenoy, 1995; Wang & Smith, 1994). Pinch is defined as the most constrained point in the process. The proposed approach determines an aggregate plan taking the pinch into consideration through graphical representation. At the pinch, the material flows in a supply chain are balanced and problem decomposition is possible. The method helps in setting targets, i.e., predicting optimal performances based on fundamental principles prior to actual scheduling of processes.

2. Graphical representation: time versus material quantity

The power of pinch analysis (Linnhoff, 1993) lies in the physical insight it provides by graphically plotting ‘quality’ versus ‘quantity’. For heat recovery systems (Linnhoff, Townsend, Boland, Hewitt, Thomas, Guy, & Marsland, 1982; Shenoy, 1995; Shenoy, Sinha, & Bandyopadhyay, 1998; Bandyopadhyay, Malik, & Shenoy, 1998, 1999), the quality is temperature while the quantity is heat duty or enthalpy content. For mass transfer systems (El-Halwagi & Manousiouthakis, 1989; Wang & Smith, 1994; Hallale & Fraser, 2000), the quality is concentration while the quantity is mass load or mass flow. For supply chains, the quality is

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time while the quantity is amount of material (mass, volume or number of units). The analogy is powerful, while the graphical representation is intuitive and relatively easy to interpret.

Material flows, material holdup and time form three important indicators of a supply chain. Pinch analysis elegantly handles these variables by plotting demand and production composites on a time versus material quantity plot (Singhvi, 2002). During aggregate planning (required to service demand in a time interval $\Delta t = t_k - t_{k-1}$), some of the decision variables are:

P_k: cumulative in-house production (number of units) at time t_k;
C_k: cumulative number of units outsourced (subcontracted) at time t_k;
D_k: cumulative demand (number of units) at time t_k as per demand forecast;
I_k: inventory at time t_k;
p_k: production rate (i.e., in-house production during the period $t_{k-1} \leq t \leq t_k$ in time Δt);
c_k: outsourced amount during the period $t_{k-1} \leq t \leq t_k$;
d_k: demand rate (i.e., demand during the period $t_{k-1} \leq t \leq t_k$ in time Δt) as per demand forecast.

A simple balance of the flow of materials at time t_k in a particular stage of the supply chain with I_0 as the initial inventory can be written as

$$I_0 + P_k + C_k = D_k + I_k \quad (1)$$

In an analogous manner, a material balance over a time interval Δt yields

$$I_{k-1} + p_k\,\Delta t + c_k = d_k\,\Delta t + I_k \quad (2)$$

It must be noted that both inventory and stockout cannot occur in the same time period; therefore, stockout can be simply viewed as negative inventory. Fig. 1 shows how Eq. (2) can be elegantly captured through typical composite curves used in pinch analysis.

Some of the salient features of the composite curves are listed below.

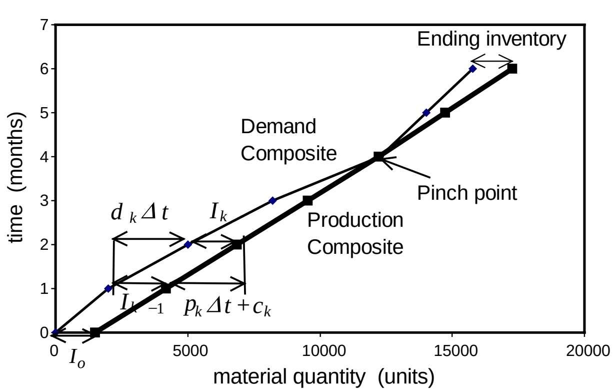

Fig. 1. Typical composites by pinch analysis.

- The demand composite curve $D(t)$ is simply a plot of the cumulative demand as a function of time, and needs to be matched by a supply composite curve $P(t)$. The demand has to be met by supply of products, some by in-house production and the rest by outsourcing. This is based on the fundamental principle of material balance.
- The vertical difference between the demand and supply composites is the lead time. Here, it is the time interval between producing an order and servicing the demand. Lead time can, in general, include the time consumed in various (supply, production, and distribution) components of the supply chain like processing and transportation. There is a lower limit T to the lead time. The point at which $P(t-T) = D(t)$ is the pinch. The two composites are separated by the minimum lead time at the pinch. In that sense, minimum lead time is analogous to the minimum temperature driving force ($\Delta T_{\min}$) in heat recovery systems. When $T = 0$, the pinch will be the point where $P(t) = D(t)$.
- The horizontal distance between the two composites at any given time gives the total inventory in the system. This also includes the work in process (WIP). The pinch is defined as the point of minimum inventory. Strictly speaking, the area between the two composites gives the measure of inventory in the system, which when multiplied by the inventory holding cost factor provides the actual inventory costs.
- A linear composite assumes constant and continuous demand or supply in a given time period. The demand composite will be a series of step functions, if actual demand has to be met at definite time intervals, and the corresponding linear composite for continuous servicing of demand will then depict the limiting case and provide the lower bound.

3. Planning for single product scenario

The pinch analysis approach is illustrated for the single product scenario using data of an example from Chopra and Meindl (2001). The demand for the product is seasonal and the company has the option to hire and lay-off workers, outsource some of the work, and build up inventory or backlogs. The company sells the product at US$ 40 per unit, but plans to give a discount of US$ 1 per unit in April. Table 1 shows the demand forecast.

Table 1
Demand data for single product case study

Time period (k)	Month	Forecasted demand (units)
1	January	1600
2	February	3000
3	March	3200
4	April	5060
5	May	1760
6	June	1760