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A stochastic framework for clearing of reactive power market

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ABSTRACT

This paper presents a new stochastic framework for clearing of day-ahead reactive power market. The uncertainty of generating units in the form of system contingencies are considered in the reactive power market-clearing procedure by the stochastic model in two steps. The Monte-Carlo Simulation (MCS) is first used to generate random scenarios. Then, in the second step, the stochastic market-clearing procedure is implemented as a series of deterministic optimization problems (scenarios) including non-contingent scenario and different post-contingency states. In each of these deterministic optimization problems, the objective function is total payment function (TPF) of generators which refers to the payment paid to the generators for their reactive power compensation. The effectiveness of the proposed model is examined based on the IEEE 24-bus Reliability Test System (IEEE 24-bus RTS).

1. Introduction

Reactive power is tightly related to bus voltages throughout a power network, and hence it has a significant effect on system security. One of the main reasons for some of recently major blackouts in the power systems around the world such as those occurred in September 23, 2003 in Sweden and Denmark, September 28, 2003 in Italy and also the United State and Canada blackout (August 2003) was reported as insufficient reactive power of system resulting in the voltage collapse [1–3]. In recent years, some papers are published in the area of optimal pricing of reactive power [4–9]. All of these papers assume that the consumer of reactive power should pay for the reactive power support service and the producers of reactive power are remunerated. Some of more recent research works on designing reactive power market also consider technical issues of the power system in addition to economical aspects [10–13]. Zhong et al. have designed a competitive reactive power market [14–17]. The other research works [18–21], took into account voltage security in the reactive power pricing. In Ref. [19], a two-level framework is proposed for the operation of a competitive reactive power market taking into account system security aspects. The first level, i.e. procurement, is on a seasonal basis while the second level, i.e. dispatch, is close to real-time operation. However, the seasonal reactive power market encounters problems discussed in Refs. [20,21]. So, these two works proposed day-ahead reactive power market instead of long term based reactive power market.

The power system has a stochastic behavior in practical operations due to uncertainties in the availability of generation, load, and transmission equipment [22]. For generating units, the uncertainties are caused by unplanned outages, equipment failures, protective relaying, economic factors including fuel prices and market prices, reserve availability, reactive power requirements, climactic variables such as precipitation and hydro-power availability, and environmental regulations and emissions restrictions. The renewable sources such as wind, photovoltaic, fuel cells, and gas micro turbines will have even more randomness than traditionally generation sources. For transmission system, the uncertainties are caused by line ratings, environmental factors such as ambient temperature and lightning, unplanned outages and equipment failures. For loads, uncertainties are caused by weather-related factors including temperature and precipitation, economic growth, new types of electronically-controlled loads, and variations in load power factors.

In the area of energy and reserve market, [23,24] have proposed a mixed integer, linear stochastic programming model for the joint dispatch of energy and reserves considering system reserve requirements based on the pre-selected set of contingencies. In the area of reactive power market however, few research works take into account the uncertain factors of electricity market. In Ref. [25] a reactive power capacity market is proposed wherein the unavailability of generators and network lines outage are considered based on a list of pre-determined most probable scenarios. In another work, a stochastic optimization model is proposed for reactive power planning in deregulated environment, assuming that the

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generator outputs and load demands can be modeled as specified probability distribution functions [26]. Nevertheless, reactive power market deserves to be cleared in such a way that the stochastic behavior of the power system is considered. For this reason, this paper proposes a stochastic framework for reactive power market clearing, taking into account generator contingencies.

The main contribution of this paper with respect to Refs. [20] and [21] and the others' works in the area is presenting a stochastic framework for clearing of day-ahead reactive power market. The uncertainty of generating units in the form of system contingencies are considered in the proposed stochastic framework for reactive power market clearing. A two-step procedure is also proposed to implement the stochastic model.

The remainder of this paper is organized as follows: In Section 2, the problem of deterministic reactive power market clearing is formulated in the form of a Mixed Integer Non-Linear Programming (MINLP) problem. In next section, the deterministic MINLP formulation is extended to the stochastic framework considering generating units' uncertainties. In Section 4, the validity of the proposed stochastic reactive power market-clearing scheme is studied based on the IEEE 24-bus RTS. Some relevant conclusions are drawn in the Section 5.

2. Deterministic reactive power market

In this section, at first, generator Expected Payment Function (EPF), proposed in Ref. [15], is reviewed in brief. The reactive power capability curve of a generator is shown in Fig. 1 [15]. Q_{base} is the reactive power required by the generator for its auxiliary equipment. If the operating point lie inside the limiting curve, e.g. (P_A, Q_{base}), then the unit can increase its reactive generation from Q_{base} to Q_A without requiring adjustment of P_A. However, this will result in the increased loss of winding and, hence, increase the cost of loss. If the generator operates on the limiting curve (field current limit), any increase in Q will require a decrease in P to adhere to the winding heating limit. Consider the operating point "A" on the curve defined by (P_A, Q_A). If more reactive power is required from the unit, for example Q_B, the operating point requires shifting back along the curve to point (P_B, Q_B), where $P_B < P_A$. This indicates that the unit has to reduce its active power output to adhere to the field heating limits when higher reactive power is demanded. The lost in the revenue to the generator due to the reduced production of active power is termed lost opportunity cost and is a significant issue in reactive power pricing.

Based on the above explanation, three operating regions for a generator on the reactive power coordinate can be defined. In region-I (0 to Q_{base}), the provided reactive power of generator is necessary for the generator own requirements to maintain its auxiliary equipment. Therefore, the generated reactive power in this region is not considered as an ancillary service to be remunerated nor the generator entitled to payments. In region-II (Q_{base} to Q_A) and ($Q_{\min}$ to 0), because of generating or absorbing reactive power, losses of generator increase and therefore, it can expect to be paid for its service. Thus, the EPF, besides availability component, will contain the cost of loss component. Finally in region-III (Q_A to Q_B), the generator is managed to reduce its active power to generate the required reactive power. Thus, the generator incurs loss of revenue cost and consequently, the EPF will contain all components of cost (availability cost, cost of loss and opportunity cost). Accordingly, the EPF can be determined in any operating condition of synchronous generator. Fig. 2 illustrates the EPF of a generator as a function of the amount of generator reactive power production [15]. According to the classification of reactive power production cost, an offer structure is formulated mathematically in Ref. [15] as the following equation:

$$\text{EPF}_i = a_{0,i} + \int_{Q_{\text{Min}}}^{0} m_{1i}\cdot \text{d}Q_i + \int_{Q_{\text{base}}}^{Q_A} m_{2i}\cdot \text{d}Q_i + \int_{Q_A}^{Q_B} (m_{3i}\cdot Q_i)\cdot \text{d}Q_i \qquad (1)$$

The coefficients in (1) represent the various components of reactive power cost incurred by the *ith* provider that need to be offered in the market where a_0 is availability price offer in dollars, m_1 is cost of loss price offer for operating in under excited mode ($Q_{\text{Min}} < Q \le 0$) in \$/MVAr-h, m_2 is cost of loss price offer for operating in region ($Q_{\text{base}} \le Q \le Q_A$) in \$/MVAr-h and m_3 is opportunity price offer for operating in region ($Q_A \le Q \le Q_B$) in \$/MVAr-h/MVAr-h (Fig. 2). $a_{0,i}$, $m_{1,i}$, $m_{2,i}$, and $m_{3,i}$ are the bid values of the *ith* provider for the reactive power market. As shown in Fig. 2, the opportunity cost is a quadratic function of Q. As shown in Refs. [14,15], the opportunity offer in region III is proportional to the amount of reactive power output (m_3Q in (1)). So, the corresponding component of EPF, i.e. $\int_{Q_A}^{Q_B} m_{3i}\cdot Q_i \cdot \text{d}Q_i$ becomes a quadratic function of Q. In other words, the payment for reactive power, apart from the cost function of the unit, is modeled as a quadratic function of Q in region III and as a linear function of Q in the regions I and II as shown in Fig. 2 and discussed in Refs. [14,15].

The reactive power market is cleared based on the minimization of total payment to the participants of the market. In other words, the objective function of this minimization problem is sum of the EPF of synchronous generator as well as condensers that should be minimized. Therefore, the total payment will depend on the market price of the four components of the reactive power compensation costs offered by the producers. The total payment function (TPF) is mathematically formulated as follows.

$$\begin{aligned}\text{TPF} = &\sum_{i=1}^{NB}\sum_{u=1}^{NU_i}\Big(\rho_0 W_{0(i,u)} - \rho_1 W_{1(i,u)}\, Q_{1G(i,u)} + \rho_2 W_{2(i,u)}\Big(Q_{2G(i,u)} \\ &- Q_{\text{base}G(i,u)}\Big) + \rho_2 W_{3(i,u)}\Big(Q_{3G(i,u)} - Q_{\text{base}G(i,u)}\Big) \\ &+ \frac{1}{2}\rho_3 W_{3(i,u)}\Big(\Big(Q_{3G(i,u)}\Big)^2 - \Big(Q_{AG(i,u)}\Big)^2\Big)\Big) \qquad (2)\end{aligned}$$

where NB is number of buses with synchronous generator or condenser and NU_i is number of units connected to the *ith* bus;

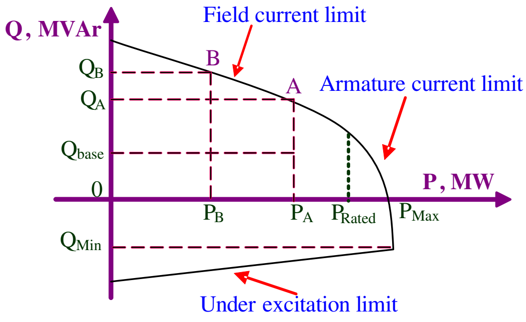

Fig. 1. Synchronous generator capability curve.

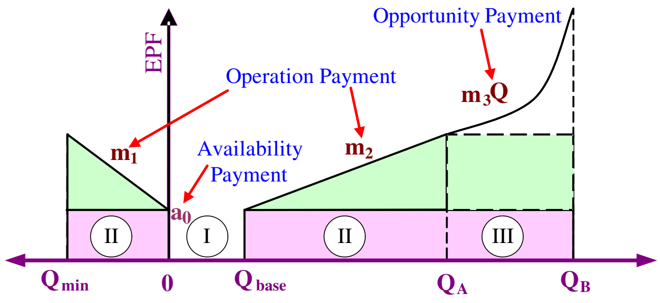

Fig. 2. Reactive power offer structure of provider.