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# An application of hybrid heuristic method to solve concurrent transmission network expansion and reactive power planning

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ABSTRACT

In this paper a mathematical model for solving simultaneous transmission network expansion and reactive power planning problem (TEPRPP) via an AC model has been presented. A Real Genetic Algorithm (RGA) combined an Interior Point Method (IPM) aimed to obtaining a significant quality solution for such a problem has been employed. The proposed algorithm is tested on three systems; IEEE 24-bus system, 46-bus south Brazilian and the Southeast Network of Iran (SNI). The obtained results show the capability and the viability of the proposed methodology incorporating the AC model of the TEPRPP problem even in real world.



## 1. Introduction

Nowadays, modern electric power systems consist of large-scale and extremely complex interconnected transmission networks. As electricity consumption grows rapidly, new transmission lines are necessary to provide alternative paths for power transfer from power plants to load centers to enable continuous supply. On the other hand, reactive power sources are desired for: increasing the capacity of transmission lines, power factor correction, loss reduction, and voltage profile improvement. Thus, Transmission Expansion Planning (TEP) and Reactive power planning (RPP) are crucial issues especially in modern power systems. By using the AC model for TEP, RPP can also be exercised. One may argue that without supporting reactive power in some transmission networks, load bus voltages may differ from their prescribed magnitudes; which may not only cause unacceptable power quality but also increase real power losses. In such cases it may require more transmission line additions which may not be economical for the network. On the other hand, TEP and RPP, as a combinatorial problem, seem very complicated and require to be solved via mixed integer nonlinear programming. The objective of the simultaneous transmission expansion and reactive power planning problem, referred to as TEPRPP, is to determine 'where', 'what' and 'when' new devices such as transmission lines and reactive power sources must be added to an existing network in order to make its operation viable for a pre-defined horizon of planning at minimum total costs. In a static TEP the time of new lines installation is not determined on the planning horizon while in a dynamic TEP in addition to "what" and "where", planners answer to "when" the transmission facilities must be installed. Since the dimension of a dynamic TEP problem is larger the solution approaches might be complicated and need huge computational effort [1]. The benchmark network of the base year, the candidate lines and candidate load buses to install reactive sources, the power generation and power demand in a planning horizon associated with the investment constraints are the basic data for such a general TEP problem. In fact, such a problem has been researched for a long time and a review of the literature can be found in [2], while the earlier well cited work is developed by Garver [3]. Albeit most of these studies only consider simplified DC models while recently an accurate AC network modeling has been proposed [4]. Generally in TEP steady-state analysis is usually performed using simplified models like: transportation models, linearized power flow models or other similar techniques. Transportation models, hybrid models, the linear disjunctive model, the DC model [5], among others, have been used to achieve the primary topology in the first stage. Solving a TEP problem can be handled both by classical optimization techniques [6,7] as well as meta-heuristics, such as: Simulated Annealing [8], Genetic Algorithms [9,10], Tabu Search [11] Greedy Randomized Adaptive Search Procedure (GRASP) [12], Ant Colony [13] and Differential Evolution [14]. In all cases, usually in a subsequent stage, the obtained expansion plan is checked for other operational constraints. In short-term planning, the steady-state studies use an AC model in order to assess real-power losses accurately and reactive compensation requirements, both for the basic configuration as

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well as contingencies. The use of the complete AC model in the first phase is incipient but there are few technical literatures on the subject [4,15].

In this paper, on the other hand, an integrated AC TEPRPP is introduced, in which the following advantages have been realized:

- Efficiently allocating reactive power sources during the planning and consequently decreasing the cost of new lines installation.
- Using an integrated mathematical model that allows TEP and the optimal allocation of reactive power simultaneously in a unique stage via and AC model.
- Incorporating the determination of the transmission system's precise real power losses in a trivial approach.

As it can be seen, in [16] the authors use a constructive heuristics algorithm to solve AC-TEP and introduce two indexes for finding weak buses for the potential location of reactive power sources. In fact, after transmission lines are constructed then reactive power sources will be allocated to weak buses. In the present work, a Real Genetic Algorithm (RGA) is used to solve TEPRPP via an AC model, while an Interior Point Method (IPM) is employed to solve an NLP problem that should be solved during the solution step of RGA. In fact, IPM provides a better computational performance for large scale problems than classical approaches, such as the Simplex method. It is worth noting that in the literature, IPM has mostly been used to solve problems like optimal power flow for large-scale systems [17], state estimation [18] and security constrained economic dispatch [19].

## 2. TEPRPP mathematical modeling

A mathematical model for the TEPRPP problem can be formulated via the following equations.

$$\min \ v_0 = \boldsymbol{c}^T \boldsymbol{n} \tag{1}$$

$$\min \ v_1 = f(\boldsymbol{q}, \boldsymbol{u}) \tag{2}$$

s.t.

$$\boldsymbol{P}(\boldsymbol{V}, \theta, \boldsymbol{n}) - \boldsymbol{P}_G + \boldsymbol{P}_D = 0 \tag{3}$$

$$\boldsymbol{Q}(\boldsymbol{V}, \theta, \boldsymbol{n}) - \boldsymbol{Q}_G + \boldsymbol{Q}_D - \boldsymbol{q} = \boldsymbol{0} \tag{4}$$

$$\underline{\boldsymbol{P}}_G \leqslant \boldsymbol{P}_G \leqslant \overline{\boldsymbol{P}_G} \tag{5}$$

$$\underline{\boldsymbol{Q}}_G \leqslant \boldsymbol{Q}_G \leqslant \overline{\boldsymbol{Q}_G} \tag{6}$$

$$\underline{\boldsymbol{q}} \leqslant \boldsymbol{q} \leqslant \bar{\boldsymbol{q}} \tag{7}$$

$$\underline{\boldsymbol{V}} \leqslant \boldsymbol{V} \leqslant \overline{\boldsymbol{V}} \tag{8}$$

$$(\boldsymbol{N} + \boldsymbol{N}^0)\boldsymbol{S}^{\text{from}} \leqslant (\boldsymbol{N} + \boldsymbol{N}^0)\overline{\boldsymbol{S}} \tag{9}$$

$$(\boldsymbol{N} + \boldsymbol{N}^0)\boldsymbol{S}^{\text{to}} \leqslant (\boldsymbol{N} + \boldsymbol{N}^0)\overline{\boldsymbol{S}} \tag{10}$$

$$0 \leqslant \boldsymbol{n} \leqslant \bar{\boldsymbol{n}} \tag{11}$$

where $\boldsymbol{c}$ and $\boldsymbol{n}$ represent the circuit costs' vector and the added lines' vector, respectively. $\boldsymbol{N}$ and $\boldsymbol{N}^0$ are diagonal matrices containing the integer-valued vector $\boldsymbol{n}$ and the existing circuits in the base configuration, respectively. $f(\boldsymbol{q},\boldsymbol{u})$ Is the cost function of reactive power (VAr) sources, $\boldsymbol{q}$ is the MVAr size of VAr sources vector. $\boldsymbol{u}$ is the binary vector that indicates whether to install reactive power sources at load buses or not and has an integer value. $v_0$ is the investment due to adding new circuits to the network and $v_1$ is the costs of VAr sources. $\bar{\boldsymbol{n}}$ is a vector containing the maximum number of circuits that can be added. $\theta$ is the unbounded phase angle vector, while $\boldsymbol{P}_G$ and $\boldsymbol{Q}_G$ are the existing real and reactive power generating vectors. Similarly, $\boldsymbol{P}_D$ and $\boldsymbol{Q}_D$ are the real and reactive power demand vectors; $\boldsymbol{V}$ is the voltage magnitude vector; $\overline{\boldsymbol{P}}_G, \overline{\boldsymbol{Q}}_G$ and $\overline{\boldsymbol{V}}$ are the vectors of maximum real and reactive power generating limits and voltage magnitudes, respectively; and $\underline{\boldsymbol{P}}_G, \underline{\boldsymbol{Q}}_G$ and $\underline{\boldsymbol{V}}$ are the vectors of minimum real and reactive power generating limits and voltage magnitudes. In this paper 105% and 95% of the nominal value are used for the maximum and minimum voltage magnitude limits, respectively. $\boldsymbol{S}^{\text{from}}, \boldsymbol{S}^{\text{to}}$ and $\boldsymbol{S}$ are the apparent power flow vectors (MVA) through the branches in both terminals and their limits. The first objective function considers only the expansion costs of transmission lines while the second objective function considers the minimum costs of VAr sources that might be installed. The limits for real and reactive power in the generators are expressed by Eqs. (5) and (6) respectively; and for VAr sources by Eq. (7) while voltage magnitudes are restricted by Eq. (8). Capacity limits (MVA) of the line flows are presented by Eqs. (9) and (10), while capacity constraints of the newly added circuits are shown by Eq. (11). The costs of VAr sources can be defined as follows:

$$f(\boldsymbol{q}, \boldsymbol{u}) = \sum_{k \in \boldsymbol{\Omega}_l} (c_{0k} + c_{1k} q_k) u_k \tag{12}$$

where $k \in \Omega_l$ represents the load buses, $\Omega_l$ is the set of all load buses; and $c_{0k}$ and $c_{1k}$ are the installation costs and unit costs for a VAr source at bus$k$. $q_k$ is the MVAr size of a VAr source installed at bus $k$ and $u_k$ is a binary variable that indicates whether to install reactive power source at bus $k$ or not. Eqs. (3) and (4) represent the conventional equations of AC power flow considering$\boldsymbol{n}$, the number of circuits (lines and transformers), and $\boldsymbol{q}$, the size of VAr sources treated as variables. The elements of vectors $\boldsymbol{P}(\boldsymbol{V}, \theta, \boldsymbol{n})$ and $\boldsymbol{Q}(\boldsymbol{V}, \theta, \boldsymbol{n})$ are calculated by Eqs. (13) and (14), respectively.

$$P_i(V, \theta, n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \cos\theta_{ij} + B_{ij}(n) \sin\theta_{ij}] \tag{13}$$

$$Q_i(V, \theta, n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \sin\theta_{ij} - B_{ij}(n) \cos\theta_{ij}] \tag{14}$$

where $i,j \in N_B$ represent buses and $N_B$ is the set of all buses, $ij$ represents the circuit between buses $i$ and $j$ and $\theta_{ij} = \theta_i - \theta_j$ is the difference in phase angle between buses $i$ and $j$. The elements of the bus admittance matrix ($\boldsymbol{G}$ and $\boldsymbol{B}$) are:

$$\boldsymbol{G} = \left\{ \begin{array}{c} G_{ij}(\boldsymbol{n}) = -\left(n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0\right) \\ G_{ii}(\boldsymbol{n}) = \sum_{j \in \Omega_l} \left(n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0\right) \end{array} \right\} \tag{15}$$

$$\boldsymbol{B} = \left\{ \begin{array}{c} B_{ij}(\boldsymbol{n}) = -\left(n_{ij} b_{ij} + n_{ij}^0 b_{ij}^0\right) \\ B_{ii}(\boldsymbol{n}) = b_i^{sh} + \sum_{j \in \Omega_l} \left[ n_{ij}\left(b_{ij} + b_{ij}^{sh}\right) + n_{ij}^0 \left(b_{ij}^0 + (b_{ij}^{sh})^0\right) \right] \end{array} \right\} \tag{16}$$

Here, $g_{ij}, b_{ij}$ and $b_{ij}^{sh}$ are the conductance, susceptance and shunt susceptance of the transmission line or transformer $ij$ (if $ij$ is a transformer $b_{ij}^{sh} = 0$), respectively; and $b_i^{sh}$ is the shunt susceptance at bus $i$, while the proposed model does not consider the phase shifters. Elements ($ij$) of vectors $\boldsymbol{S}^{\text{from}}$ and $\boldsymbol{S}^{\text{to}}$ of (9) and (10) are given by the following relationship:

$$S_{ij}^{from} = \sqrt{\left(P_{ij}^{from}\right)^2 + \left(Q_{ij}^{from}\right)^2} \tag{17}$$

$$P_{ij}^{from} = V_i^2 g_{ij} - V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}) \tag{18}$$

$$Q_{ij}^{from} = -V_i^2 \left(b_{ij}^{sh} + b_{ij}\right) - V_i V_j (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij}) \tag{19}$$

$$S_{ij}^{to} = \sqrt{\left(P_{ij}^{to}\right)^2 + \left(Q_{ij}^{to}\right)^2} \tag{20}$$

$$P_{ij}^{to} = V_j^2 g_{ij} - V_i V_j (g_{ij} \cos\theta_{ij} - b_{ij} \sin\theta_{ij}) \tag{21}$$

$$Q_{ij}^{to} = -V_j^2 \left(b_{ij}^{sh} + b_{ij}\right) + V_i V_j (g_{ij} \sin\theta_{ij} + b_{ij} \cos\theta_{ij}) \tag{22}$$