Int. J. Production Economics 121 (2009) 68–71

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An integrated production planning model for molds and end items

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ARTICLE INFO

Article history:
Received 8 November 2007
Accepted 23 September 2008
Available online 14 April 2009

Keywords:
Serial production system
Molds
Production planning
Preventive maintenance
Nonlinear stochastic programming model

ABSTRACT

This study develops a mathematical modelling framework for simultaneously generating production plans for molds and the end items that are made with them. The inputs considered are the item demand (assumed constant over an infinite planning horizon), holding costs and shortage costs, together with the molds' statistical lifetime distribution (in terms of number of uses) and costs pertaining to amortization, preventive replacements and corrective replacements.

1. Introduction

A company producing souvenirs and ornaments (hereafter called *end items*) out of a semi-precious metal wanted to improve its production planning capability, but doing so was hampered by the fact that the rubber molds in which the molten metal is poured are subject to random failure. Numerous reasons account for the variability in the molds' lifetime, some of which are labor-related (for instance the care with which a mold is opened and an item is released), and others that are due to factors such as quality of raw materials and pre-heat temperature. This paper proposes a nonlinear stochastic optimization framework for jointly establishing, under the assumption of a single item with constant demand over an infinite planning horizon, the production rates of the items and the molds. The objective function to be minimized is comprised of mold amortization costs, preventive replacement costs and failure replacement costs as well as item holding costs and shortage costs. Although mold shortages may also occur, the resulting costs are assumed to be entirely reflected in the ensuing shortages of end items.

Note that the decision structure considered in this paper is traditionally split up, both in practice and in the literature, with different objectives in mind. That is, a production plan for end items would be hatched in view of fulfilling customer orders, following which a production plan for molds would be devised considering the mold availability and the end item production plan. By contrast, a mathematical programming model integrating both decisions is developed in Section 2, with concluding remarks given in Section 3.

We now give a brief literature review from the combined production and maintenance realm. Van der Dyun Schouten and Vanneste (1995) proposed a preventive maintenance policy based on the age of the machine and the capacity of a buffer stock for a production line consisting of two machines. Meller and Kim (1996) studied the impact of preventive maintenance on a system with two machines and a fixed buffer stock capacity between the machines. Srinivasan and Lee (1996) analyzed the combined effects of preventive maintenance and production/inventory policies on the operating costs of the production unit. For their part, Cheung and Hausmann (1997) proposed the simultaneous optimization of stock and an age-type maintenance policy. Boukas and Haurie (1990), Gharbi and Kenne (2000) and Kenne and Gharbi (2001) consider the ordering of the production flow and preventive maintenance by using a Markov

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0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2008.09.016

model. Sarker and Haque (2000) carried out a simulation of a production system with a random failure rate in order to analyze jointly spare parts provisioning and maintenance. Iravani and Duenyas (2002) presented an integrated maintenance/repair and production/inventory model using a Markov decision process. Rezg et al. (2004) presented the joint optimization of preventive maintenance and stock control on a production line made up of N machines. Kenne and Gharbi (2004) studied the stochastic optimization of a problem of production control with corrective and preventive maintenance. They propose a method to find the optimal age of preventive maintenance and production rates for a production system composed of identical machines. In Aghezzaf et al. (2007) the objective is to determine an integrated production and maintenance plan that minimizes the expected total production and maintenance costs over a finite planning horizon. Meantime, Panagiotidou and Tagaras (2006) developed a model for the optimization of preventive maintenance procedures in a production process that may operate in one of two different quality states and is also subject to failure.

Cormier and Rezg (2007) examined a somewhat similar problem to that studied here, but estimated the cost per period by means of simulation for a set of production rates. Based on this output, a cost function was then formulated via experimental design, which finally allowed an optimum to be determined analytically. This method is less precise than the one presented in the present paper, but has the advantage of yielding results even when the (probability density function) pdf of the mold's lifetime is not known analytically. Let us now proceed with the mathematical formulation for the problem at hand.

2. A mathematical programming model for planning the production of molds and end items

In this section, we formulate a nonlinear stochastic programming model for simultaneously establishing the production rates of molds and end items. Note that the probability density function of the lifetime of the molds in terms of number of uses is specified in continuous form as this tends to be common in the preventive maintenance literature and also allows us to develop a mathematical programming model in continuous variables (desirable for computational efficiency). The error potentially introduced owing to the discrete nature of the number of uses would be small since the mean of the distribution is assumed to be large. We also point out that the molds are not repairable and are in the as-good-as-new condition upon being replaced. All item shortages are assumed lost.

2.1. Parameters

- α the cost of producing a mold
- C_p the cost of a preventive replacement of a mold
- C_f the cost of a failure replacement of a mold
- h the holding cost per period, per unit of the end item
- s the shortage cost per unit of the end item
- D the end item's per-period demand
- p the per-period production capacity of the end item
- y the number of uses of a mold until it fails (we denote this as the mold's lifetime)
- $f(y)$ the probability density function (pdf) of y, $y \geqslant 0$
- w the number of periods required to replace a mold that has failed while producing the end items
- $\overline{w}$ the expectation of w
- x the number of periods required to preventively replace a mold
- $\overline{x}$ the expectation of x

2.2. Variables

- N the number of times a mold is used until it is preventively replaced
- n the per-period number of uses of a mold
- L the average length of a mold cycle (in terms of number of periods), defined as the mold's life expectancy plus the time required to replace it either preventively or following a failure
- γ^+ the average inventory level of the end item
- γ^- the average per-period shortage of the end item

2.3. Model formulation

Following the logic of optimal age-based preventive replacement of a machine (in the present case, the mold) subject to breakdown (see e.g., Jardine and Tsang, 2006), note that a cycle may give rise to either a preventive replacement or a failure replacement. Let $R(N)$ denote the reliability function (that is, the probability of a mold surviving N uses, equal to $\int_N^\infty f(y)dy$), and $F(N) = 1 - R(N)$. Then the mean time to failure of a mold, given that it does not survive N uses, is equal to $\int_0^N yf(y)dy/F(N)$. Hence, the average length of a preventive cycle, plus the average length of a failure cycle (not including the time required to carry out the replacements), in terms of number of uses, is equal to $NR(N) + [\int_0^N yf(y)dy/F(N)]F(N)$. Therefore, $L = \frac{1}{n}[NR(N) + \int_0^N yf(y)dy] + \overline{x}R(N) + \overline{w}F(N)$, the last two terms respectively corresponding to the portion of an average mold cycle spent replacing a mold preventively or following a failure.

Now, calculating the inventory level requires that the average mold cycle be divided into three contiguous mutually exclusive segments, namely, $k = 1$: $t \in (0, \overline{w}F(N)]$, $k = 2$: $t \in (\overline{w}F(N), L - \overline{x}R(N)]$, and $k = 3$: $t \in (L - \overline{x}R(N), L]$. Let $I_{t,k}$ denote the inventory level at time t, with t in segment k, and assume that $I_{0,1} = 0$ (otherwise inventory would accumulate indefinitely since the inventory at the end of one mold cycle is the inventory at the beginning of the next). Observe that the inventory level will peak at a certain time, following which it will decrease to 0 at say time T, $T < L$ (see Fig. 1).

The probability of a mold being available at $t \in (0, \overline{w}F(N)]$, given that a failure occurring any time prior to $\overline{w}F(N)$ would not leave sufficient time to replace the mold before t, is $R(tn)$ (tn being the number of times the mold is used over t periods). Hence,

$$I_{t,1} = n \int_0^t R(yn)dy - tD \; ; \; t \in (0, \overline{w}F(N)] \tag{1}$$

Meantime, the probability of a mold being available at $t \in (\overline{w}F(N), L - \overline{x}R(N)]$ is the conditional probability that the mold survives tn uses given that it survived $[t - \overline{w}F(N)]n$