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Optimal reactive power flow via the modified barrier Lagrangian function approach

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ABSTRACT

A new approach called the Modified Barrier Lagrangian Function (MBLF) to solve the Optimal Reactive Power Flow problem is presented. In this approach, the inequality constraints are treated by the Modified Barrier Function (MBF) method, which has a finite convergence property; i.e. the optimal solution in the MBF method can actually be in the bound of the feasible set. Hence, the inequality constraints can be precisely equal to zero. Another property of the MBF method is that the barrier parameter does not need to be driven to zero to attain the solution. Therefore, the conditioning of the involved Hessian matrix is greatly enhanced. In order to show this, a comparative analysis of the numeric conditioning of the Hessian matrix of the MBLF approach, by the decomposition in singular values, is carried out. The feasibility of the proposed approach is also demonstrated with comparative tests to Interior Point Method (IPM) using various IEEE test systems and two networks derived from Brazilian generation/transmission system. The results show that the MBLF method is computationally more attractive than the IPM in terms of speed, number of iterations and numerical conditioning.

1. Introduction

The Optimal Power Flow (OPF) is one of the most powerful tools to analyse static systems of electrical energy. There are countless studies concerning the OPF which depend on the interest of the network operator. It is the ideal tool for a deregulation environment in which one can obtain, among others, the spot price for a composition of tariffs, an optimal dispatch of generators and synchronous condensers and static VAr, or even the adjustment of any device for the desired performance of the network. Due to this motivation, the OPF has been intensely studied and important progress has taken place since it was proposed.

In the past three decades, various optimization techniques, such as nonlinear and quadratic programming, Newton-based, linear programming and IPM were proposed to solve the nonlinear OPF problem shown in [1,2]. Among earlier techniques applied to solving the OPF problem, the IPM is considered to be efficient, mainly due to its good performance and ease of handling inequality constraints. The first implementation of the IPM applied to the OPF problem was proposed in [3]. In the same year, the Primal-dual Interior Point algorithm with the Predictor-Corrector method [4] was used to accelerate the convergence of the problem. In recent years, practically all research involving the OPF problem has been based on the variants of the IPM [5–12] and some researches have been based in different approaches [13–16].

Despite all of this progress, the IPM and its variants are based on the Logarithmic Barrier Function (LBF), therefore these methods have inherent drawbacks, for example, along the trajectory that approaches the solution the barrier parameter has been driven to zero, which makes the Hessian matrix of the LBF become increasingly ill-conditioned. However, the LBF improved more when Polyak [17] introduced the Modified Barrier Functions (MBF) and corresponding methods.

In the same work, Polyak showed the characteristics of the MBF. This function and its derivate are defined in the solution. They do not grow infinitely, the barrier parameter does not need to be driven to zero and the Hessian matrix of the MBF does not become ill-conditioned when the current approximation approaches the solution. In addition, the MBF method converges with a fixed barrier parameter as opposed to an asymptotic one for methods based on the LBF. It also allows constraints to become precisely equal to zero, thus including the boundary of the original optimization problem in its feasible region, whereas in the LBF, the solution can only be close, but never reaching the boundaries. Unlike the methods based on LBF, the MBF method does not require any feasible solution, for equality and inequality constraints, as a starting point. Another interesting characteristic of the MBF is the explicit

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representation of its Lagrange multiplier, which helps the convergence of the method.

The first applications of the MBF method and its variants to the OPF problem were reported in [18,19]. In [18] the modified barrier-augmented Lagrangian method [20], a variant of MBF method, was used for the optimum selections of the transformers tap positions and the voltage points of the generators. This approach treated the inequality constraints using the MBF without the slack variables, and equality constraints by the augmented Lagrangian function. The feasibility of this approach was demonstrated using a 160-bus test system. Another application of MBF method was to establish the pricing mechanism for finding the equilibrium in an auction market [19], i.e., finding such prices for the goods and services that the optimal choices of the consumers lead to a balance between supply and demand.

With the aim of furthering studies related to solving the OPF problem and motivated by the efficiency of the MBF method, in this paper we propose a new approach, MBLF, based on this method to solve the Optimal Reactive Power Flow problem. The feasibility of the proposed approach is demonstrated using comparative tests with the IPM [3,5] and IPM with the Predictor-Corrector (IPM-PC) [4,5] using several system.

This paper is organized as follows: in Section 2 the MBLF method is presented; in Section 3 the tests and results are shown, in Section 4 conclusions are drawn, and finally in the appendix the development of the MBF is described.

2. Modified barrier Lagrangian function method

In the MBLF method, the bounded constraints are transformed into two inequalities. Slack variables are introduced, transforming these inequalities into equalities. The slack variables are relaxed and treated by the MBF. A Lagrangian is associated with the problem. The first-order necessary conditions are applied to this function, generating a system of nonlinear equations, whose roots are calculated by Newton's method.

The Optimal Reactive Power Flow (ORPF) problem is a nonlinear programming problem, which can be represented as:

$$\begin{aligned} &\text{Minimize} && f(\boldsymbol{x}) \\ &\text{subject to} && \mathbf{g}(\boldsymbol{x}) = 0 \\ & && \underline{\boldsymbol{h}} \le \boldsymbol{h}(x) \le \bar{\boldsymbol{h}} \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in R^n$ is the control and state variable vector. The scalar $f(\boldsymbol{x})$ is the objective function. The vector function $g(\boldsymbol{x}) \in R^m$, where $m < n$, is the equality constraints and the vector function $\boldsymbol{h}(\boldsymbol{x}) \in R^p$, with lower bound $\underline{h}$ and upper bound $\bar{\boldsymbol{h}}$, is the inequality constraints.

The ORPF problem can be solved by the MBLF approach, in which the positive slack variables are introduced to transform the inequality constraints into the equality ones.

$$\begin{aligned} &\text{Minimize} && f(\boldsymbol{x}) \\ &\text{subject to} && g(\boldsymbol{x}) = 0 \\ & && \boldsymbol{h}(\boldsymbol{x}) + s_1 = \bar{\boldsymbol{h}} \\ & && \boldsymbol{h}(\boldsymbol{x}) - s_2 = \underline{\boldsymbol{h}} \\ & && \boldsymbol{s}_1 \ge 0 \\ & && \boldsymbol{s}_2 \ge 0 \end{aligned} \tag{2}$$

where the slack vectors $\boldsymbol{s}_1 \in R^p$ and $\boldsymbol{s}_2 \in R^p$.

The non-negative conditions of problem (2) are relaxed by the barrier parameter. This represents an expansion of the feasible region of the original problem.

$$\begin{aligned} &\text{Minimize} && f(\boldsymbol{x}) \\ &\text{subject to} && \boldsymbol{g}(\boldsymbol{x}) = 0 \\ & && \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_1 = \bar{\boldsymbol{h}} \\ & && \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_2 = \underline{\boldsymbol{h}} \\ & && \boldsymbol{s}_1 \ge -\mu \\ & && \boldsymbol{s}_2 \ge -\mu \end{aligned} \tag{3}$$

where $\mu > 0$ is the barrier parameter.

The MBF, proposed by [17], is used to transform problem (3) into the following modified problem. The development of MBF is presented in the Appendix.

$$\begin{aligned} &\text{Minimize} && f(\boldsymbol{x}) \\ &\text{subject to} && \boldsymbol{g}(\boldsymbol{x}) = 0 \\ & && \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_1 = \bar{\boldsymbol{h}} \\ & && \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_2 = \underline{\boldsymbol{h}} \\ & && \mu \ \ln(\mu^{-1}\boldsymbol{s}_1 + 1) \ge 0 \\ & && \mu \ \ln(\mu^{-1}\boldsymbol{s}_2 + 1) \ge 0 \end{aligned} \tag{4}$$

The following Lagrangian function is associated to problem (4).

$$L = f(\boldsymbol{x}) - \mu \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{u}_{1\boldsymbol{i}} \ln(\mu^{-1}\boldsymbol{s}_{1\boldsymbol{i}} + 1) - \mu \sum_{\boldsymbol{j}=1}^{\boldsymbol{p}} \boldsymbol{u}_{2\boldsymbol{j}} \ln(\mu^{-1}\boldsymbol{s}_{2\boldsymbol{j}} + 1)$$
$$- \boldsymbol{\lambda}^{\boldsymbol{T}}\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{\pi}_1(\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_1 - \bar{\boldsymbol{h}}) - \boldsymbol{\pi}_2(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_2 - \underline{\boldsymbol{h}}) \tag{5}$$

where $\boldsymbol{u}_1 \in R^p$, $\boldsymbol{u}_2 \in R^p$, $\boldsymbol{\lambda} \in R^m$, $\boldsymbol{\pi}_1 \in R^p$ and $\boldsymbol{\pi}_2 \in R^p$ are the vectors of the Lagrange multipliers. Function (5) is called the modified barrier Lagrangian function. The first-order necessary conditions are applied to the modified barrier Lagrangian function, generating nonlinear system equations:

$$\nabla_{\boldsymbol{d}}\boldsymbol{L}(\boldsymbol{x}, \boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{\lambda}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2) = 0 \tag{6}$$

where

$$\boldsymbol{d}^{\boldsymbol{T}} = (\boldsymbol{x}, \boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{\lambda}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$$

$$\nabla_{\boldsymbol{d}}\boldsymbol{L} = \begin{bmatrix} \nabla_{\boldsymbol{x}} f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\boldsymbol{T}}\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{\pi}_1^{\boldsymbol{T}}\boldsymbol{J}_1(\boldsymbol{x}) - \boldsymbol{\pi}_2^{\boldsymbol{T}}\boldsymbol{J}_1(\boldsymbol{x}) \\ -\dfrac{\boldsymbol{u}_1}{\mu^{-1}\boldsymbol{s}_1 + 1} + \boldsymbol{\pi}_1 \\ -\dfrac{\boldsymbol{u}_2}{\mu^{-1}\boldsymbol{s}_2 + 1} - \boldsymbol{\pi}_2 \\ -\boldsymbol{g}(\boldsymbol{x}) \\ -(\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_1 - \bar{\boldsymbol{h}}) \\ -(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_2 - \underline{\boldsymbol{h}}) \end{bmatrix} \tag{7}$$

with: $\boldsymbol{J}(\boldsymbol{x})^{\boldsymbol{T}} = (\nabla_{\boldsymbol{x}} \boldsymbol{g}_1(\boldsymbol{x}), \ldots, \nabla_{\boldsymbol{x}} \boldsymbol{g}_{\boldsymbol{m}}(\boldsymbol{x}))$, and $\boldsymbol{J}_1(\boldsymbol{x})^{\boldsymbol{T}} = (\nabla_{\boldsymbol{x}} \boldsymbol{h}_1(\boldsymbol{x}), \nabla_{\boldsymbol{x}} \boldsymbol{h}_2(\boldsymbol{x}), \ldots, \nabla_{\boldsymbol{x}} \boldsymbol{h}_{\boldsymbol{p}}(\boldsymbol{x}))$, which are the Jacobian matrices.

Newton's method is applied to the nonlinear system Eq. (6) to find the search direction vector $\Delta\boldsymbol{d}$. The application of Newton's method results in linear system equations, which can be represented by

$$\boldsymbol{W}\Delta\boldsymbol{d} = -\nabla_{\boldsymbol{d}}\boldsymbol{L} \tag{8}$$

where:

$$\Delta\boldsymbol{d}^{\boldsymbol{T}} = (\Delta\boldsymbol{x}, \Delta\boldsymbol{s}_1, \Delta\boldsymbol{s}_2, \Delta\boldsymbol{\lambda}, \Delta\boldsymbol{\pi}_1, \Delta\boldsymbol{\pi}_2)$$