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An approximation-based approach for fuzzy multi-period production planning problem with credibility objective

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A B S T R A C T

This paper develops a fuzzy multi-period production planning and sourcing problem with credibility objective, in which a manufacturer has a number of plants or subcontractors. According to the credibility service levels set by customers in advance, the manufacturer has to satisfy different product demands. In the proposed production problem, production cost, inventory cost and product demands are uncertain and characterized by fuzzy variables. The problem is to determine when and how many products are manufactured so as to maximize the credibility of the fuzzy costs not exceeding a given allowable invested capital, and this credibility can be regarded as the investment risk criteria in fuzzy decision systems. In the case when the fuzzy parameters are mutually independent gamma distributions, we can turn the service level constraints into their equivalent deterministic forms. However, in this situation the exact analytical expression for the credibility objective is unavailable, thus conventional optimization algorithms cannot be used to solve our production planning problems. To overcome this obstacle, we adopt an approximation scheme to compute the credibility objective, and deal with the convergence about the computational method. Furthermore, we develop two heuristic solution methods. The first is a combination of the approximation method and a particle swarm optimization (PSO) algorithm, and the second is a hybrid algorithm by integrating the approximation method, a neural network (NN), and the PSO algorithm. Finally, we consider one 6-product source, 6-period production planning problem, and compare the effectiveness of two algorithms via numerical experiments.

1. Introduction

Production planning is viewed as the plans and arrangements of the production mission and progress in production scheduled time. In recent years, uncertain production planning has received much attention in the field of production planning management, where uncertainty can be present as randomness and fuzziness in the production environments. This uncertainty will result in more realistic production planning models. However, the inclusion of uncertainty in the production system parameters is a more difficult task in terms of modeling and solving. Over the years, there has been much research and many applications with the aim of modeling the uncertainty in the production planning problems, including material requirements planning models [1,2], hierarchical production planning models [3,4], aggregate production planning model [5], supply chain models [6,7], and other well-known production planning models in the literature [8].

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To handle probabilistic uncertainty in production decision systems, some meaningful stochastic production planning models have been documented in the literature. Bitran [9] dealt with a stochastic production planning problem with a service level requirement and provided non-sequential and deterministic equivalent formulations of the model; Schmidt [10] presented a Markov decision process model that combines features of engineering design models and aggregate production planning models to obtain a hybrid model that links biological and engineering parameters to optimize operations performance; Yildrim et al. [11] studied a stochastic multi-period production planning and sourcing problem of a manufacturer with a number of plants or subcontractors and presented a methodology that a manufacturer can utilize to make its production and sourcing decisions, and Kelly et al. [12] extended the economic lot scheduling problem for the single-machine multi-product case with random demands, their objective was to find the optimal length of production cycles that minimizes the sum of set-up costs and inventory holding costs per unit of time and satisfies the demand of products at the required service levels.

On the other hand, with the development of fuzzy set and possibility theories [13–15], a number of researchers realized the importance to handle possibilistic uncertainty in decision systems, and applied the fuzzy theory to various production planning problems. Wang and Fang [16] presented a fuzzy linear programming model for solving the aggregate production planning problem with multiple objectives; Inuiguchi et al. [17] discussed the merits of possibilistic programming approach to production planning problems and applied a possibilistic programming based on possibility and necessity measures to solve the production planning problem. The reader who is interested in related issues in this field may refer to [18–20].

Since the credibility of fuzzy event and the expected value of fuzzy variable was defined in [21], an axiomatic approach called credibility theory has been developed in recent years (see [22,23]). Some interesting applications about credibility in production decision systems have been studied in the literature such as [24,25]. Maity et al. [24] proposed an optimal control approach to optimizing the production, recycling and disposal strategy so that the total expected profit is maximized, and Mandal et al. [25] developed an optimal production inventory model with fuzzy time period and fuzzy inventory costs for defective items and solved it under fuzzy space constraint. In the current development, we take credibility theory as the theoretical foundation of fuzzy optimization and formulate a novel class of multi-period production planning problems with credibility objective, in which product demands, production and inventory costs are uncertain and characterized by fuzzy variables. The objective of the problem is to maximize the credibility of the fuzzy costs not exceeding a given allowable invested capital, and this credibility can be regarded as the investment risk criteria under fuzzy environment. In general, the credibility functions in the service level constraints are difficult to compute, so we discuss the cases when demands are independent gamma fuzzy variables. In this situation, we can transform the credibility service level constraints to their equivalent deterministic forms. However, the analytical expression about the credibility objective is unavailable, and the equivalent production planning problem is neither linear nor convex, thus conventional optimization algorithms cannot be employed to solve it. Therefore, two heuristic solution methods are designed to solve the proposed production planning model. The first is the PSO algorithm (see [26,27]) combining with the approximation method [28], and the second is the hybrid PSO algorithm by integrating both the approximation method and an NN. One 6-product source, 6-period production planning problem is provided to compare the effectiveness of the designed algorithms.

The plan of this paper is as follows. Section 2 formulates a new class of fuzzy production planning models. In Section 3, we discuss the computation for the credibility objective of the production planning model, and deal with the convergence of computational method. In addition, due to the complexity of the proposed production planning problem, two heuristic solution methods are designed in this section. The first is the PSO algorithm based on the approximation method, and the second is a hybrid algorithm by integrating the approximation method, an NN and the PSO algorithm. Section 4 presents one 6-product source, 6-period production planning problem to compare the effectiveness of two algorithms. Section 5 concludes the paper.

2. Problem formulation

In this section, we will construct a new type of fuzzy programming models for multi-period production planning and sourcing problems with fuzzy parameters. The characteristic of this manufacturing system can be summarized as follows.

- There are N types of production sources (plants and subcontractors) in the system, and the decision of production levels to meet market demands must be taken for T periods. The demand in each period is not known with certainty and characterized by a fuzzy variable.
- The costs that are used in the objective function of the model consist of production cost and inventory carrying cost. The production and inventory cost coefficients are not known exactly and represented by fuzzy variables. In general, we assume fuzzy demands, production and inventory cost coefficients in different periods are mutually independent (see [29]).
- Constraints on the performance (related to backorders) of the system are imposed by requiring service levels which force the credibility of having no stock out to be greater than or equal to a predetermined service level requirement in each period.