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A fuzzy control system with application to production planning problems

Zhongfeng Qin [a], Manying Bai [a,*], Dan Ralescu [b]

[a] School of Economics and Management, Beihang University, Beijing 100191, China
[b] Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221-0025, USA

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ABSTRACT

A considerable part of the literature on fuzzy sets is devoted to the field of fuzzy control system. In this paper, an alternative control system is introduced to describe a dynamic system with fuzzy white noise. In order to find optimal ways to control such a system, fuzzy optimal control theory is further developed. Specifically, a linear quadratic model is formulated and solved as a fuzzy optimal control problem. The formulation and solution of this model provide an economic interpretation of a production planning model both in the finite horizon and in the infinite horizon.

1. Introduction

Classical control systems are expressed by differential equations. However, uncertainty is inherent in most dynamic systems. In order to characterize a system with white noise, stochastic control systems were introduced via Ito's stochastic differential equation. Stochastic control theory is developed to find optimal ways to control such a system, and has been widely applied to physical, biological, economic and management problems.

It is well known that fuzziness is another kind of uncertainty present in real systems. Fuzzy set theory [17] has been applied for modeling control systems in different ways such as in Zadeh [18–20]. Traditionally, these models use *If-Then* rules and logical connectives to establish relations between the system variables. Depending on the form of the propositions and on the structure of the rule, different types of rule-based fuzzy models can be distinguished. See Sousa and Kaymak [14] and references therein for a review of these models, and several new developments [3,4].

In the past, two basic types of fuzzy control systems have been studied. They are the Mamdani fuzzy control system [10] and the Takagi–Sugeno fuzzy control system [15]. Each of these fuzzy control systems can be used to produce rule-based models for deterministic systems.

An alternative approach, *the Liu fuzzy control system*, introduced in this paper, is that of a fuzzy control system driven by a Liu process [7]. Unlike the Mamdani and Takagi–Sugeno systems, the Liu fuzzy control system is not deterministic. Instead, it is characterized by a *fuzzy differential equation*. Based on this, a linear quadratic model is proposed and the corresponding fuzzy optimal control problem is solved. Finally, the system is applied to model production planning problems.

The remainder of this paper is organized as follows: Section 2 recalls some basic concepts of Liu fuzzy control system. The proposed fuzzy optimal control problem is introduced and some results are given in Section 3. The linear quadratic model is formulated and the general solution procedure is obtained in Section 4. In Section 5, a type of production planning model in

* Corresponding author.
E-mail addresses: qin@buaa.edu.cn, qzf05@mails.tsinghua.edu.cn (Z. Qin), baimy@buaa.edu.cn (M. Bai).

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the finite horizon is formulated and solved, and some economic interpretations are given. Section 6 considered the infinite horizon case of production planning model. The related works are shown in Section 7, and the comparisons with stochastic production planning model and the limitations of our approach are discussed in Section 8. Finally, some conclusions of our work are listed.

2. The Liu fuzzy control system

The concept of fuzzy set was introduced by Zadeh [17] via membership functions. In order to define a self-dual measure for fuzzy events, the concept of credibility measure was proposed by Liu and Liu [9], and was further refined by Liu [8] to study the behavior of fuzzy phenomena. In addition, a sufficient and necessary condition for credibility measure was given by Li and Liu [6]. The change of fuzziness in time is studied by fuzzy calculus [7] using the concepts of fuzzy differential equation [7] as a type of differential equation driven by a Liu process. Detailed expositions of fuzzy calculus and fuzzy differential equation are found in [11,12].

Definition 1 (*Liu [7]*). A fuzzy process C_t is said to be a Liu process if:

(i) $C_0 = 0$.
(ii) C_t has stationary and independent increments.
(iii) Every increment $C_{t+s} - C_s$ is a normally distributed fuzzy variable with expected value et and variance $\sigma^2 t^2$ and whose membership function is

$$\mu(x) = 2\left(1 + \exp\left(\frac{\pi|x - et|}{\sqrt{6}\sigma t}\right)\right)^{-1}, \quad -\infty < x < \infty.$$

The process is in standard form if $e = 0$ and $\sigma = 1$.

A deterministic control system is governed by a differential equation, where the state variables of the system are known with certainty. However, it may not be possible to measure the state of the system over time because of perturbations of white noise. Thus, it is reasonable to assume that the state follows a stochastic process. In many cases, there is no historical data and thus the corresponding probability distributions are not known. Therefore, it is difficult or impossible to deal with white noise. Alternatively, the state equation can be assumed to have a fuzzy white noise. A Liu fuzzy control system is defined to capture this situation as follows:

Definition 2. A Liu fuzzy control system is a system characterized by a special fuzzy differential equation

$$\mathrm{d}X_t = f(t, X_t, Z_t)\mathrm{d}t + g(t, X_t, Z_t)\mathrm{d}C_t, \tag{1}$$

where C_t is a standard Liu process, X_t is the state variable, and Z_t is a control.

Remark 1. If the diffusion g is identically zero, then (1) degenerates into a deterministic control system.

Remark 2. If C_t is replaced by Brownian motion, then (1) describes a traditional stochastic control system.

Example 1. A linear Liu fuzzy control system is described by a linear fuzzy differential equation

$$\mathrm{d}X_t = (\alpha X_t + \beta Z_t + e)\mathrm{d}t + (\gamma X_t + \delta Z_t + \sigma)\mathrm{d}C_t, \tag{2}$$

where α, β, γ, δ, e and σ are real numbers.

Example 2. Consider an asset that pays a perpetual cash flow at a rate of $X_t\mathrm{d}t$, where X_t follows a Liu process with drift Z_t and volatility σ. That is to say, the system is governed by the linear fuzzy differential equation

$$\mathrm{d}X_t = Z_t\mathrm{d}t + \sigma X_t\mathrm{d}C_t.$$

3. The fuzzy optimal control problem

In general, a criterion is associated with a system and an optimization model is formulated to optimize the criterion for Liu fuzzy control system. That is to say, some objective functionals are given to measure the performance of this system. A variety of fuzzy optimal control models may be formulated to characterize the practical problems according to different objective functionals. The aim of a fuzzy optimal control model is to choose the best control Z_t such that some given objective functional related to X_t is optimized.