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Decomposition–coordination interior point method and its application to multi-area optimal reactive power flow

Wei Yan [a,*], Lili Wen [b], W. Li [c], C.Y. Chung [a,d], K.P. Wong [d]

[a] State Key Laboratory of Power Transmission Equipment & System Security and New Technology, College of Electrical Engineering, Chongqing University, Chongqing 400030, China
[b] Test & Research Institute of Chongqing Electric Power, Chongqing 401123, China
[c] British Columbia Transmission Corporation, Vancouver, BC, Canada V7X 1V5
[d] Department of Electrical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

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ABSTRACT

A decomposition–coordination interior point method (DIPM) is presented and applied to the multi-area optimal reactive power flow (ORPF) problem in this paper. In the method, the area distributed ORPF problem is first formed by introducing duplicated border variables. Then the nonlinear primal dual interior point method (IPM) is directly applied to the distributed ORPF problem in which a Newton system with border-matrix-blocks is formulated. Finally the overall ORPF problem is solved in decomposition iterations with the Newton system being decoupled. The proposed DIPM inherits the good performance of the traditional IPM with a feature appropriate for distributed calculations among multiple areas. It can be easily extended to other distributed optimization problems of power systems. Numeric results of five IEEE Test Systems are demonstrated and comparisons are made with those obtained using the traditional auxiliary problem principle (APP) method. The results show that the DIPM for the multi-area OPRF problem requires less iterations and CPU time, has better stability in convergence, and reaches better optimality compared to the traditional auxiliary problem principle method.

1. Introduction

A modern power system is an interconnected grid and contains widely distributed sub-networks each of which represents an independent utility. Generally, each utility company has its own operation criteria and separate EMS. Local optimal simulations in the individual EMSs will produce great errors. Therefore, there is a need of coordinating solutions of the individual EMSs to achieve the overall optimal solution. Decomposition techniques in a distributed computing environment have attracted great attention.

Many different decomposition techniques have been proposed during the past forty years, such as Dantzig-Wolfe technique [1,2], Lagrangian relaxation technique [3], augmented Lagrangian technique [4–7], and approximate Newton directions [9,10]. Particularly remarkable is the theoretical decomposition framework based on the auxiliary problem principle (APP). It has been used widely in solving many operation problems of power systems, such as the daily generation scheduling [4], distributed state estimation [5], multi-area optimal power flow [6,7], and multi-area ORPF [8]. But the APP method presents only modest speed-ups and efficiencies even in ideal situations [6], and sometimes leads to poor convergence when related parameters are selected improperly [10].

This paper proposes a new decomposition method based on the nonlinear primal dual interior point method (IPM) [11–15]. Similar to the APP method, the proposed method also uses the concept of duplicated border variables to implement an area decomposition of the original overall problem. Differently, in the proposed method, the objective function, variables and constraints are structured in such a way that a Newton system with border-matrix-blocks is created. This Newton system is then decoupled into a set of linear subsystems so that the overall problem can be solved through separate computations of linear area subsystems in the Newton iteration process of IPM. The proposed method only computes the linear subsystems corresponding to individual areas and avoids resolutions of optimization sub-problems with respective iteration processes. Thus, the proposed method has an advantage in computational efficiency superior to the APP and other Lagrangian relaxation or augmented Lagrangian decomposition algorithms while it inherits good performance of the IPM in fast convergence. This is because the proposed method only needs the decoupling implementation of the Newton system in the IPM.

The proposed decomposition–coordination interior point method (DIPM) is applied to solve the multi-area optimal reactive power flow (ORPF) in the paper. It should be pointed out that it can also be extended to other distributed computing problems of power sys-

* Corresponding author.
E-mail addresses: cquyanwei@cqu.edu.cn (W. Yan), wll563003@sina.com (L. Wen), wen.yuan.li@bctc.com (W. Li), eecychun@polyu.edu.hk (C.Y. Chung), eekpwong@polyu.edu.hk (K.P. Wong).

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tems. The rest of the paper is organized as follows. Section 2 provides the formulation of area decentralization model of ORPF. Section 3 describes the proposed decomposition method. In Section 4, numerical examples are given to demonstrate the effectiveness of the proposed method, followed by Section 5 for conclusions.

2. Area decentralization model of ORPF

2.1. Centralized ORPF formulation

The formulation of the original ORPF problem can be mathematically expressed as the following minimization nonlinear programming problem:

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ s.t. \quad & \boldsymbol{g}\boldsymbol{x} = 0 \\ & \underline{\boldsymbol{x}} \leqslant \boldsymbol{x} \leqslant \bar{\boldsymbol{x}} \end{aligned} \tag{1}$$

where the objective function $f(\boldsymbol{x})$ is total active power losses; $\boldsymbol{g}(\boldsymbol{x})$ is the nonlinear vector function representing power flow equations; $\boldsymbol{x}$ is the vector of state and control variables, including voltage magnitudes and angles at load buses, injected reactive powers of generators, voltage magnitudes at generator buses, reactive powers of shunt capacitors/reactors and transformer tap ratios; $\bar{\boldsymbol{x}}$ and $\underline{\boldsymbol{x}}$ are the vectors representing operational limits on state and control variables.

2.2. Area-based decentralization for ORPF

The decomposition idea has been described in [6]. The basic approach is to divide the overall system into geographical areas. Any transmission line that crosses between two adjacent areas is divided into two sections by adding a "dummy bus" at the border between the two areas. Real and reactive power flow variables and voltage magnitude and angle variables are defined for the dummy bus and these four border variables are duplicated, with one copy assigned to each area. To be consistent with the overall formulation, any pair of corresponding duplicated variables in the two areas has the exactly same value.

The decomposition method of an interconnected power system can be explained using Fig. 1. The system consists of two areas as shown in Fig. 1a. Area-1 and area-2 are connected by the border bus B. $\boldsymbol{x}_{I1}$ and $\boldsymbol{x}_{I2}$ denote the internal variables belonging to each area. $\boldsymbol{x}_B$ is the border variables of bus B, which includes its voltage magnitude, voltage angle, active and reactive powers transferred along the tie-line. By splitting bus B into two duplicated dummy buses B_1 and B_2, as shown in Fig. 1b, two separated systems are obtained with each one having a dummy bus. At the same time, $\boldsymbol{x}_B$ is also duplicated as $\boldsymbol{x}_{B1}$ and $\boldsymbol{x}_{B2}$ which are assigned to area-1 and area-2 respectively.

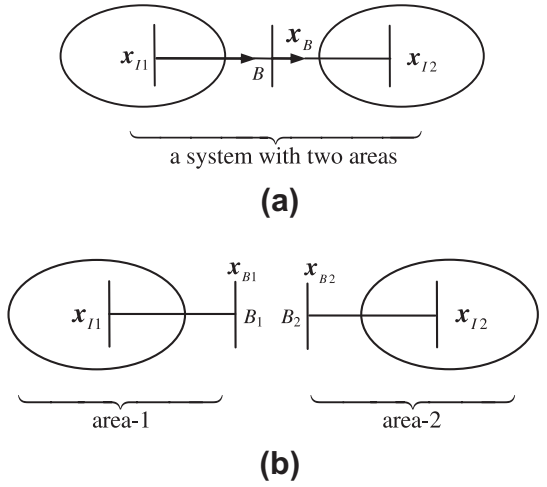

Fig. 1. Decomposition of an interconnected power system.

With such duplication of border variables, problem (1) is fully equivalent to the following expression:

$$\begin{aligned} \min \quad & f_1(\boldsymbol{x}_{I1}, \boldsymbol{x}_{B1}) + f_2(\boldsymbol{x}_{I2}, \boldsymbol{x}_{B2}) \\ s.t. \quad & \boldsymbol{g}_1(\boldsymbol{x}_{I1}, \boldsymbol{x}_{B1}) = 0 \\ & \boldsymbol{g}_2(\boldsymbol{x}_{I2}, \boldsymbol{x}_{B2}) = 0 \\ & \underline{\boldsymbol{x}}_{I1} \leqslant \boldsymbol{x}_{I1} \leqslant \bar{\boldsymbol{x}}_{I1} \\ & \underline{\boldsymbol{x}}_{I2} \leqslant \boldsymbol{x}_{I2} \leqslant \bar{\boldsymbol{x}}_{I2} \\ & \underline{\boldsymbol{x}}_{B1} \leqslant \boldsymbol{x}_{B1} \leqslant \bar{\boldsymbol{x}}_{B1} \\ & \underline{\boldsymbol{x}}_{B2} \leqslant \boldsymbol{x}_{B2} \leqslant \bar{\boldsymbol{x}}_{B2} \\ & \boldsymbol{x}_{B1} - \boldsymbol{x}_{B2} = 0 \end{aligned} \tag{2}$$

where $f_i(\boldsymbol{x}_{Ii}, \boldsymbol{x}_{Bi})$ represents the objective function of each subsystem or area-i, whose equality constraints are denoted by $\boldsymbol{g}_i(\boldsymbol{x}_{Ii}, \boldsymbol{x}_{Bi}) = 0$. Equation $\boldsymbol{x}_{B1} - \boldsymbol{x}_{B2} = 0$ represents the condition that any pair of duplicated variables in the two areas has the exactly same value.

Formulation (2) is the area-based decentralization model of ORPF. Apparently, formulation (2) can be extended to a system with N areas. By denoting $\boldsymbol{x}_i$ for $(\boldsymbol{x}_{Ii}, \boldsymbol{x}_{Bi})^T$ $(i = 1, 2)$ and $\boldsymbol{A}_i$ for the coefficient matrix representing the coupled border condition, a general formulation for the multiple-area model of ORPF can be expressed as follows:

$$\begin{aligned} \min \quad & F = \sum_{i=1}^{N} f_i(\boldsymbol{x}_i) \\ s.t. \quad & \boldsymbol{g}_i(\boldsymbol{x}_i) = 0 \\ & \underline{\boldsymbol{x}}_i \leqslant \boldsymbol{x}_i \leqslant \bar{\boldsymbol{x}}_i \\ & \sum_i \boldsymbol{A}_i \boldsymbol{x}_i = 0, \quad i = 1, \ldots, N \end{aligned} \tag{3}$$

It can be seen from formulation (3) that the objective function and all the constraints except for the coupled border condition can be divided into N independent sub-problems, and the variables in each sub-problem are associated with ones in other sub-problems only through the coupled border constraints.

3. Decomposition–coordination IPM

In this section, the decomposition–coordination interior point method (DIPM) for the distributed ORPF problem is presented. The key idea is the construction of a border-matrix-block and the decoupling of the Newton system in applying the IPM to problem (3).

3.1. IPM for ORPF

The ORPF problem shown in formulation (3) can be solved directly using the IPM. In this method, slack variables and Lagrange multipliers are introduced to deal with inequality and equality constraints, and logarithmic barrier functions are used to guarantee the non-negativity condition of slack variables. Then the ORPF problem given in the formulation (3) is transformed into one non-constrained optimization problem with the following Lagrange function:

$$\begin{aligned} L = & \sum_{i=1}^{N} f_i(\boldsymbol{x}_i) - \sum_{i=1}^{N} \boldsymbol{y}_i^T \boldsymbol{g}_i(\boldsymbol{x}_i) - \sum_{i=1}^{N} \boldsymbol{z}_i^T (\boldsymbol{x}_i - \boldsymbol{l}_i - \underline{\boldsymbol{x}}_i) + \sum_{i=1}^{N} \boldsymbol{w}_i^T (\boldsymbol{x}_i \\ & + \boldsymbol{u}_i - \bar{\boldsymbol{x}}_i) - \boldsymbol{y}_d^T \left[\sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i \right] \\ & - \sum_{i=1}^{N} \mu_i \left[\sum_{j=1}^{n_i} \ln(l_{i(j)}) + \sum_{j=1}^{n_i} \ln(u_{i(j)}) \right] \end{aligned} \tag{4}$$