Computer Vision and Image Understanding 113 (2009) 802–810

Contents lists available at ScienceDirect

Computer Vision and Image Understanding

journal homepage: www.elsevier.com/locate/cviu

A new energy-based method for 3D motion estimation of incompressible PIV flows

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ARTICLE INFO

Article history:
Received 14 December 2007
Accepted 28 January 2009
Available online 6 February 2009

Keywords:
Particle Image Velocimetry
Optical flow
Partial differential equation
Variational methods
Incompressible flows

ABSTRACT

Motion estimation has many applications in fluid analysis, and a lot of work has been carried out using Particle Image Velocimetry (PIV) to capture and measure the flow motion from sequences of 2D images. Recent technological advances allow capturing 3D PIV sequences of moving particles. In the context of 3D flow motion, the assumption of incompressibility is an important physical property that is satisfied by a large class of problems and experiments. Standard motion estimation techniques in computer vision do not take into account the physical constraints of the flow, which is a very interesting and challenging problem. In this paper, we propose a new variational motion estimation technique which includes the incompressibility of the flow as a constraint to the minimization problem. We analyze, from a theoretical point of view, the influence of this constraint and we design a new numerical algorithm for motion estimation which enforces it. The performance of the proposed technique is evaluated from numerical experiments on synthetic and real data.

1. Introduction

Particle Image Velocimetry (PIV) is a technique which aims at obtaining image sequences of fluid motion in a variety of applications, in gaseous and liquid media, and at extracting the corresponding flow velocity information (see [1]). The typical setting of a PIV experiment includes the following components: the flow medium seeded with particles, droplets or bubbles, a double pulsed laser which illuminates the particles twice within a short interval of time, a light sheet optics guiding a thin light plane within the flow medium, one or several CCD cameras which capture the two frames exposed by the laser pulses and a timing controller synchronizing the laser and the camera. Standard PIV techniques estimate two planar components of the fluid motion from 2D images (2D-PIV). Using stereo techniques, dual-plane PIV or holographic recording (3C-PIV) the third spatial component of the flow can be also estimated, as in [2]. The extension of PIV techniques to full spatial domain (3D-PIV) is currently an active research area (see [3] for more details).

The most common technique for motion estimation in 2D-PIV is based on local correlation between two rectangular regions of two consecutive images, as in [4]. This technique has a straightforward extension to 3D images. Another approach to motion estimation widely used in computer vision is based on energy minimization, also called variational technique, where on the one hand, we assume the conservation of the image intensity of the scene objects across the image sequence (in PIV sequences, the scene objects are the particles); and on the other hand, we assume a certain regularity of the flow. In the context of 2D PIV, such an approach has been proposed by Corpetti et al. [5].

When dealing with 3D fluid flow estimation, sufficient information is captured to model physical constraints of the flow, such as the Navier–Stokes equations or the incompressibility. Currently, an interesting challenging problem is to include these constraints into the motion estimation model.

In this paper, we propose a new variational motion estimation technique which includes the incompressibility of the flow as a constraint in the minimization problem. In order to minimize the constrained energy, we use a generalized Lagrange multiplier approach.

The paper is organized as follows: in Section 2, we briefly describe related works on flow motion estimation, in Section 3, we present the classic Helmholtz vector field decomposition that we use to analyze the constrained energy problem. In Section 4, we propose a new variational model constrained by the incompressibility of the flow. In Section 5, we present our numerical scheme and implementation. In Section 6, the performance of the proposed method is evaluated from experimental results on synthetic and real data, before the discussion and conclusion.

2. Related works on motion estimation

Among the different existing approaches to estimate motion between two images, the methods based on local cross-correlation and the methods based on energy minimization (variational

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doi:10.1016/j.cviu.2009.01.005

approaches) are widely used in the literature. In this section, we briefly review both approaches.

2.1. Motion estimation using local cross-correlation

Cross-correlation is the most common technique for fluid motion estimation in PIV (see for instance [1,4]). We denote I_1 and I_2 two consecutive images, $\mathbf{u}$ represents the unknown flow motion between I_1 and I_2 and N is the image dimension (in our case $N = 3$). Ω is the domain of definition of the images, and $\partial\Omega$ is the boundary of Ω.

For each voxel $\mathbf{v} = (v_x, v_y, v_z) \in \Omega$, the correlation technique compares a rectangular subvolume $I_{1,\mathbf{v}}$ of I_1 centered on $\mathbf{v}$, with a subvolume of I_2 centered on a neighbor $\mathbf{v} + \mathbf{d}$ of $\mathbf{v}$. The similarity measure between two rectangular subvolumes is based on the cross-correlation and is defined as:

$$C_{\mathbf{v}}(I_1, I_2)(\mathbf{d}) = \sum_{\mathbf{y} \in I_{1,\mathbf{v}}} I_1(\mathbf{v} + \mathbf{y}) I_2(\mathbf{v} + \mathbf{d} + \mathbf{y}). \tag{1}$$

The displacement $\mathbf{d}$ which maximizes the above similarity measure provides the flow motion estimation at voxel $\mathbf{v}$.

Usually, the correlation is computed in the Fourier domain to reduce the computational cost of the cross-correlation method. The accuracy of the correlation method is improved by applying the algorithm iteratively using decreasing correlation window sizes, and sub-voxel precision is obtained by means of interpolation schemes.

2.2. Standard energy-based method

Variational approaches to motion estimation are widely used in computer vision (see for instance [6–15]). In the framework of fluid mechanics some interesting contributions can be found in [16–22].

In a variational approach, the flow motion is estimated by minimizing an energy functional. The standard energy functional $E(\mathbf{u})$, that we use in this paper, is written as:

$$E(\mathbf{u}) = \underbrace{\int_\Omega (I_1(\mathbf{x}) - I_2(\mathbf{x} + \mathbf{u}(\mathbf{x})))^2 d\mathbf{x}}_{\text{data term}} + \underbrace{\alpha \int_\Omega \|\nabla \mathbf{u}(\mathbf{x})\|^2 d\mathbf{x}}_{\text{regularization term}}, \tag{2}$$

where $\mathbf{u}(\mathbf{x}) = (u^x(\mathbf{x}), u^y(\mathbf{x}), u^z(\mathbf{x}))^t$ denotes the unknown vector flow, $\|\nabla \mathbf{u}(\mathbf{x})\|^2$ is defined as $\|\nabla u^x(\mathbf{x})\|^2 + \|\nabla u^y(\mathbf{x})\|^2 + \|\nabla u^z(\mathbf{x})\|^2$, and α is a scalar coefficient that weights the smoothing term. Under the assumption of intensity conservation for each voxel, the first term (*data term*) becomes zero when the first image matches the second one displaced by $\mathbf{u} : I_1(\mathbf{x}) = I_2(\mathbf{x} + \mathbf{u}(\mathbf{x}))$. The second term is a *regularization term* which smooths the vector field. Several regularization terms have been proposed in the literature in order to better preserve discontinuities and improve the robustness, as in [9,15]. In this paper, since we focus on the incompressibility of the estimated flow as a new constraint, for simplicity, we will use the classic regularity term presented above.

Usually, homogeneous Neumann conditions ($\mathbf{u} \cdot \mathbf{n} = 0$ in $\partial\Omega$ where $\mathbf{n}$ is the normal boundary vector field) are assumed at the boundaries. These boundary conditions minimize the boundary influence in the flow estimation. The Euler–Lagrange equations of the energy yields to the partial differential equation system:

$$\nabla E(\mathbf{u}) = (I_1(\mathbf{x}) - I_2(\mathbf{x} + \mathbf{u})) \cdot \nabla I_2(\mathbf{x} + \mathbf{u}) + \alpha \Delta \mathbf{u} = 0, \tag{3}$$

where $\Delta \mathbf{u} = (\Delta u^x, \Delta u^y, \Delta u^z)^t$. The coefficient α is usually normalized to allow invariance under global intensity change. To this purpose, α is written as

$$\alpha = \alpha' \left(\sqrt{\frac{1}{|\Omega|} \int_\Omega \|\nabla \mathbf{I_2}(\mathbf{x})\|^2 d\mathbf{x}} \right)^2, \tag{4}$$

where α' is a constant parameter. In the context of 3D PIV flow, this energy have been studied in previous work by [23,24]. In these publications, the energy of Eq. (2) is not constrained by the incompressibility of the flow. In [24], the incompressibility is obtained by applying iterative solenoidal projection of the current flow motion while minimizing the unconstrained energy. It has the important limitation that the incompressibility constraint is not included in the energy itself. In this paper, we overcome this limitation by solving the constrained energy optimization problem. This technique is based on a Lagrange multipliers generalization theorem presented in Section 4.

3. Helmholtz vector field decomposition

The result presented in this section can be found in classical books on fluid mechanics (see for instance [25–27]). First, we introduce the following notations:

$$H_s \equiv \left\{ \mathbf{u}_s \in C^1(\Omega) \cap C(\bar{\Omega}) : div(\mathbf{u}_s) = 0 \text{ in } \Omega \text{ and } \mathbf{u}_s \cdot \mathbf{n} = 0 \text{ in } \partial\Omega \right\}, \tag{5}$$

where $\mathbf{n}(\mathbf{x})$ is the vector normal to the boundary of Ω, and

$$H_r \equiv \left\{ \mathbf{u}_r \in \mathbf{C}^1(\Omega) \cap C(\bar{\Omega}) : \exists\, p \in H^1(\Omega) \text{ such that } \mathbf{u}_r = \nabla p \text{ in } \Omega \right\}, \tag{6}$$

where $H^1(\Omega)$ is the usual Sobolev space.

Theorem 1 (Helmholtz decomposition). *Let $\mathbf{u} \in C^1(\Omega) \cap C(\bar{\Omega})$ be a 3D vector field, then there exists $\mathbf{u}_s \in H_s$ and $\mathbf{u}_r \in H_r$ such as:*

$$\mathbf{u} = \mathbf{u}_s + \mathbf{u}_r,$$

and where $\mathbf{u}_s$ and $\mathbf{u}_r$ are orthogonal in $L^2(\Omega)$, i.e.

$$\int_\Omega \mathbf{u}_s(\mathbf{x}) \mathbf{u}_r(\mathbf{x}) d\mathbf{x} = 0,$$

and $\mathbf{u}_r = \nabla p$ where p is a solution of the Poisson's equation:

$$\begin{aligned} \Delta p &= div(\mathbf{u}) \quad \text{in } \Omega \\ \frac{\partial p}{\partial \mathbf{n}} &= \mathbf{u} \cdot \mathbf{n} \quad \text{in } \partial\Omega, \end{aligned} \tag{7}$$

denoting Δp the usual Laplacian operator.

We observe that the above Poisson's equation allows us to estimate the projection of $\mathbf{u}$ in H_r and H_s. Indeed, given $p(\cdot)$ the solution of the above Poisson's equation, we can define the following projection operators in the space H_r and H_s:

$$P_r(\mathbf{u}) \equiv \mathbf{u}_r = \nabla p, \tag{8}$$

$$P_s(\mathbf{u}) \equiv \mathbf{u}_s = \mathbf{u} - \nabla p. \tag{9}$$

4. Incompressibility constrained model

A large class of real fluid flows satisfies the incompressibility constraint (i.e. $div(\mathbf{u}) = 0$). In this section, we focus on how to include such an incompressibility constraint in the 3D flow motion estimation. To this end, we look for local minima of the constrained energy

$$\min_{\mathbf{u} \in H_s} E(\mathbf{u}), \tag{10}$$

where $E(\mathbf{u})$ is defined in Eq. (2) and H_s is defined in Eq. (5). Next, we show a generalization of the Lagrange multipliers technique applied to the constrained energy problem Eq. (10).

First, let us recall that the classic Lagrange multipliers technique to minimize a constrained energy $E(\mathbf{u})$, where the constraint is given by $g(\mathbf{u}) = 0$, is based on the minimization of the unconstrained energy: