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A linear optimization approach to the combined production planning model ☆

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Abstract

Two fundamental processes usually arise in the production planning of many industries. The first one consists of deciding how many final products of each type have to be produced in each period of a planning horizon, the well-known lot sizing problem. The other process consists of cutting raw materials in stock in order to produce smaller parts used in the assembly of final products, the well-studied cutting stock problem. In this paper the decision variables of these two problems are dependent of each other in order to obtain a global optimum solution. Setups that are typically present in lot sizing problems are relaxed together with integer frequencies of cutting patterns in the cutting problem. Therefore, a large scale linear optimizations problem arises, which is exactly solved by a column generated technique. It is worth noting that this new combined problem still takes the trade-off between storage costs (for final products and the parts) and trim losses (in the cutting process). We present some sets of computational tests, analyzed over three different scenarios. These

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results show that, by combining the problems and using an exact method, it is possible to obtain significant gains when compared to the usual industrial practice, which solve them in sequence.

Keywords: Lot sizing; Cutting stock; Column generation technique; Linear optimization approach

1. Introduction

Due to economical aspects and computational advances, the complexity of optimization models in industrial processes has been increasing considerably. It is still common that most research is focused on solving isolated industrial problems. But with the growth and general dissemination of computer and optimization-based planning, industries have been looking for more advanced global methods. Instead of determining optimal solutions of isolated problems, people of industries are looking for integrated solutions that represent the industrial activities more accurately. For instance, managers that have tools to solve separately logistics and production planning, now are interested in treating both problems in conjunction, to obtain a global cost minimization. Obviously, by joining various problems of high complexity, one is faced with a problem much more difficult to solve, since there are additional coupling constraints, and a global optimum solution to the combined problem is not in general a straight composition of each optimal solution of the isolated problems.

In this paper we focus on the case of furniture industries, but extensions to many others industrial settings are straightforward. Given a demand of final products (Fig. 1(c)) over a planning horizon, the issue addressed consists of deciding what to produce in each period over the planning horizon and how to arrange the parts (Fig. 1(b)) in plates (Fig. 1(a)) in order to minimize the trim loss during the cutting process. Therefore, two problems of high complexity arise in this production planning. The first one, *the lot sizing problem*, which consists of planning the quantity of each type of final product to be produced in each period. Setup costs may be associated with production decision for each final product in any given period. The second problem that arises in the case of furniture industries consists of cutting rectangular plates in order to produce smaller rectangular parts used in the assembly of final products, the well-known *rectangular guillotineable two-dimensional cutting stock problem* (see [22]). In this way, combining these two problems, the issue consists of the trade-off analysis existent when we solve the cutting stock problem taking into account the production planning for various periods. Probably it would be worth of anticipating the production of lots of parts or final products, increasing the storage costs, but reducing losses in cutting process as well as decreasing the quantity of setups. Thus, three economical factors have influence on the combined problem: the trim loss, storage and setup costs.

Fig. 1. (a) Rectangular plates to be cut, (b) rectangular parts and (c) final products.