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Minimum risk criterion for uncertain production planning problems

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ABSTRACT

This paper considers a new class of multi-product source and multi-period fuzzy random production planning problems with minimum risk and service levels where both the demands and the production costs are assumed to be uncertain and characterized as fuzzy random variables with known distributions. The proposed problems are formulated as a fuzzy random production planning (FRPP) model by maximizing the mean chance of the total costs less than a given allowable investment level. Because the exact value of the objective function for a given decision variable cannot be easily obtained, we adopt an approximation approach (AA) to evaluate the objective value and then discuss the convergence of the AA, including the convergence of the objective value, the convergence of the optimal solutions and the convergence of the optimal value. Since the approximating multi-product source multi-period FRPP model is neither linear nor convex, an approximation-based hybrid monkey algorithm (MA) which combines the AA, stochastic simulation (SS), neural network (NN) and MA is designed to solve the proposed model. Finally, numerical examples are provided to illustrate the effectiveness of the hybrid monkey algorithm.

1. Introduction

In recent years, production planning especially uncertain production planning has been studied widely in the field of production planning management. Mula, Poler, García-Sabater, and Lario (2006) presented that models for production planning which do not recognize the uncertainty could be expected to generate inferior planning decisions as compared to models that explicitly account for the uncertainty. Galbraith (1973) defined the uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. Ho (1989) categorized the uncertainties into two groups: (i) environmental uncertainties and (ii) system uncertainties. Environmental uncertainties included the uncertainties beyond the production process such as demand uncertainty and supply uncertainty. System uncertainties were related to uncertainties within the production process such as operation yield uncertainty, production lead time uncertainty and quality uncertainty.

Among the above-mentioned uncertainties, randomness and fuzziness play a pivotal role. These uncertainties will result in more realistic production planning models. In order to handle the randomness in the production decision systems, some meaningful stochastic production planning models have been proposed in the literature. For example, Tarima and Brian (2004) addressed the multi-period single-item inventory lot-sizing problem with stochastic demands at the required service levels. Lin (2009) presented a stochastic version of single-source capacitated facility location problem in which a set of capacitated facilities with service level requirements was to be selected to provide service to demand points with stochastic demand at the minimal total cost.

In fuzzy decision systems, with the development of fuzzy set theory proposed by Dubois and Prade (1988), Nahmias (1978), Zadeh (1965), Zadeh (1978), fuzzy production planning models have been considered by several researchers such as Wang and Fang (2001), Liang (2008), Tanaka, Guo, and Zimmermann (2000), Mula, Poler, and García-Sabater (2007) and Lan, Liu, and Sun (2010). Among them, Wang and Fang (2001) presented a fuzzy linear programming model for solving the aggregate production planning problems with multiple objectives. Liang (2008) developed a fuzzy multi-objective linear programming (FMOLP) model to solve multi-product and multi-time period production/distribution planning decisions (PDPD) problems. Tanaka et al. (2000) transformed possibilistic linear programming problems based on exponential possibility distributions into non-linear optimization problems. In order to solve optimization problems easily, algorithms for obtaining center vectors and distribution matrices in sequence were proposed. Mula et al. (2007) proposed a new fuzzy mathematical programming model for production planning under uncertainty in an industrial environment which considered fuzzy constraints related to the total costs, the market demand and the available capacity of the productive resources and fuzzy coefficients for the costs due to the backlog of demand. Lan et al. (2010) developed a fuzzy multi-period production planning and sourcing problem with credibility objective.

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However, in real world problems, decision makers may encounter a hybrid uncertain environment where randomness and fuzziness coexist in a decision system. In fact, the decision maker cannot obtain the perfect information of each parameter in the decision system, sometimes, the probability distribution functions of the variable may be partially known by estimation. In such environments, the random variables contain fuzzy information. Fuzzy random theory introduced by Kwakernaak (1978) is a powerful tool to deal with this twofold uncertainties. Based on fuzzy random theory, Hao and Liu (2008) recently developed two classes of mean–variance models for portfolio selection problems, in which the security returns were assumed to be characterized by fuzzy random variables. Wang and Watada (2009) discussed the analytical properties of mean chance distribution functions and critical value functions of fuzzy random variables, and obtained several useful continuity theorems, and Liu (2008) presented a new class of fuzzy random minimum-risk problems (FRMRPs) via the mean chance, and applied the FRMRP to the capacitated location-allocation problems with fuzzy random demands. Moreover, the interested readers may refer to Nematian, Eshghi, and Eshragh-Jahromi (2010) and Sun, Liu, and Lan (2010).

The purpose of this paper is to present a realistic production planning model for the multi-product source and multi-period fuzzy random production planning problems, in which both the demands and the production costs are assumed to be uncertain and characterized as fuzzy random variables with known distributions. The objective function of the proposed model is to maximize the mean chance of the total costs less than a given allowable investment level. Then, we apply an approximation approach (AA) proposed by Liu (2006) to evaluate the value of the objective function and discuss the convergence of the AA, including the convergence of the objective value, the convergence of the optimal solutions and the convergence of the optimal value. Since the approximating FRPP production planning model is neither linear nor convex, it cannot be solved via the conventional optimization algorithm. Therefore, an approximation-based hybrid monkey algorithm which combines the AA, SS, NN and MA proposed by Zhao and Tang (2008) is designed to solve the proposed model. Finally, numerical examples are provided to illustrate the effectiveness of the hybrid MA.

The rest of this paper is organized as follows. In Section 2, we recall some preliminary knowledge. Section 3 proposes a multi-product source and multi-period FRPP model. In Section 4, the AA is employed to approximate the value of the objective function of the proposed model. The convergence of the AA, including the convergence of the objective value, the convergence of the optimal solutions and the convergence of the optimal value is discussed in Section 5. The convergent result facilitates us to design an approximation-based algorithm to solve the proposed model in Section 6, and numerical examples are provided in Section 7 to illustrate the effectiveness of the hybrid MA. Section 8 summarizes the main results in this paper.

2. Preliminaries

A possibility space is defined as a triplet $(\Gamma, \mathscr{P}(\Gamma), \text{Pos})$, where Γ is a nonempty set, $\mathscr{P}(\Gamma)$ the power set of Γ, and Pos a possibility measure defined on $\mathscr{P}(\Gamma)$. If ξ is a function from Γ to $\Re$, then it is called a fuzzy variable in Nahmias (1978). The relationship between possibility measure Pos and the membership function $\mu(x)$ can be expressed as

$$\text{Pos}(\{\xi \in B\}) = \sup_{x\in B} \mu(x).$$

The credibility measure of the fuzzy event $\{\xi \in B\}$ proposed by Liu and Liu (2002) is defined as

$$\text{Cr}(\{\xi \in B\}) = \frac{1}{2}\left(1 + \sup_{x\in B}\mu(x) - \sup_{x\in B^c}\mu(x)\right).$$

It is easy to check that Cr has the following property:

$$\text{Cr}(\{\xi \in B\}) + \text{Cr}(\{\xi \in B^c\}) = 1,$$

which is referred to the self-duality.

Example 1. Let ξ be a fuzzy variable with the following membership function

$$\mu_\xi(x) = \begin{cases} 0, & \text{if } x \leqslant -1 \\ \frac{x+1}{2}, & \text{if } -1 < x \leqslant 1 \\ \exp\left(-\frac{(x-1)^2}{2}\right), & \text{if } x > 1, \end{cases}$$

which is shown in Fig. 1.

For every $r \in \Re$,

$$\text{Pos}(\{\xi \geqslant r\}) = \begin{cases} 1, & \text{if } r \leqslant 1 \\ \exp\left(-\frac{(r-1)^2}{2}\right), & \text{if } r > 1, \end{cases}$$

$$\text{Pos}(\{\xi < r\}) = \begin{cases} 0, & \text{if } r \leqslant -1 \\ \frac{r+1}{2}, & \text{if } -1 < r \leqslant 1 \\ 1, & \text{if } r > 1. \end{cases}$$

Thus, the credibility of $\{\xi \geqslant r\}$ is

$$\text{Cr}(\{\xi \geqslant r\}) = \frac{1}{2}(1 + \text{Pos}(\{\xi \geqslant r\}) - \text{Pos}(\{\xi < r\})) = \begin{cases} 1, & \text{if } r \leqslant -1 \\ \frac{1-r}{2}, & \text{if } -1 < r \leqslant 1 \\ \frac{1}{2}\exp\left(-\frac{(r-1)^2}{2}\right), & \text{if } r > 1. \end{cases}$$

Similarly,

$$\text{Cr}(\{\xi < r\}) = \begin{cases} 0, & \text{if } r \leqslant -1 \\ \frac{1+r}{2}, & \text{if } -1 < r \leqslant 1 \\ 1 - \frac{1}{2}\exp\left(-\frac{(r-1)^2}{2}\right), & \text{if } r > 1. \end{cases}$$

It is evident that for every $r \in \Re$, the equality Cr $(\{\xi \geqslant r\})$ + Cr $(\{\xi < r\}) = 1$ holds truly.

In Liu and Gao (2007), a sequence $\{\xi_n\}$ of fuzzy variables is said to be mutually independent if and only if

$$\text{Cr}(\{\xi_i \in B_i, \quad i = 1, 2, \ldots, n\}) = \min_{1\leqslant i\leqslant n} \text{Cr}(\{\xi_i \in B_i\})$$

for any subsets $B_1, B_2, \ldots, B_n$ of $\Re$ and any positive integer n.

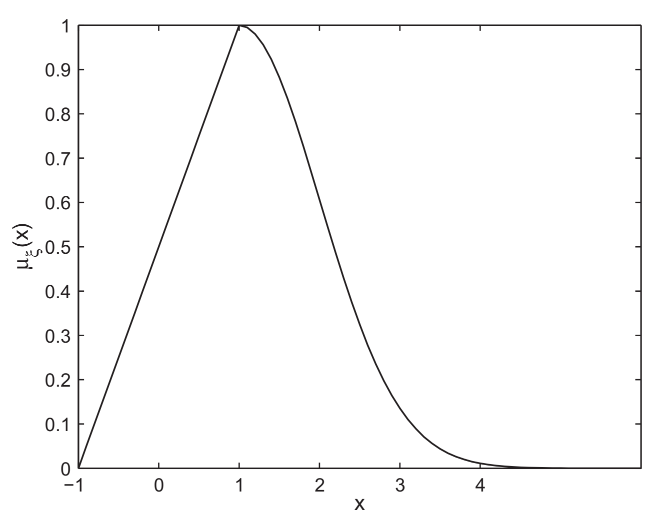

Fig. 1. The membership function of ξ.