



Temporal and spatial Lagrangean decompositions in multi-site, multi-period production planning problems with sequence-dependent changeovers

Sebastian Terrazas-Moreno^a, Philipp A. Trotter^b, Ignacio E. Grossmann^{a,*}

^a Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15232, USA

^b RWTH Aachen University, Templergraben 55, 52056 Aachen, Germany

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ABSTRACT

We address in this paper the optimization of a multi-site, multi-period, and multi-product planning problem with sequence-dependent changeovers, which is modeled as a mixed-integer linear programming (MILP) problem. Industrial instances of this problem require the planning of a number of production and distribution sites over a time span of several months. Temporal and spatial Lagrangean decomposition schemes can be useful for solving these types of large-scale production planning problems. In this paper we present a theoretical result on the relative size of the duality gap of the two decomposition alternatives. We also propose a methodology for exploiting the economic interpretation of the Lagrange multipliers to speed the convergence of numerical algorithms for solving the temporal and spatial Lagrangean duals. The proposed methods are applied to the multi-site multi-period planning problem in order to illustrate their computational effectiveness.

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1. Introduction

The optimal planning of a network of manufacturing sites and markets is a complex problem. It involves assigning which products to manufacture in each site, how much to ship to each market and how much to keep in inventory to satisfy future demand. Each site has different production capacities and operating costs, while demand for products varies significantly across markets. Production and distribution planning is concerned with mid to long-term decisions usually involving several months, adding a temporal dimension to the spatial distribution given by the multi-site network. The production of each product can involve a setup or cleaning time that in some cases is sequence-dependent. When setups and sequence-dependent transitions are included, the optimal planning problem becomes a mixed-integer linear programming (MILP) problem. The computational expense of solving large-scale MILP problems can be decreased by using decomposition techniques. This paper presents temporal and spatial Lagrangean decompositions that allow the independent solution of time periods, production sites, and markets. The importance of choosing between alternative Lagrangean relaxations of the same planning model is discussed by Gupta and Maranas (1999). Jackson and Grossmann (2003) use temporal decomposition for solving a multi-site, multi-period planning problem. They report that temporal decomposition provides a tighter bound on the

full space solution and has faster dual convergence than spatial decomposition. Wu and Ierapetritou (2006) also implement Lagrangean decomposition on a multi-period scheduling problem. These authors propose to use the Nelder–Mead approach as an alternative to subgradient method for updating the multipliers. Temporal decomposition using this approach results in a significant reduction in computational time. Neiro and Pinto (2006) use temporal Lagrangean decomposition to solve a multi-period MINLP planning problem under uncertainty concerning a petroleum refinery. They find that this decomposition scheme helps overcome the exponential increase in solution time that occurs with the full space model. In a problem similar to the one presented in this paper, Chen and Pinto (2008) use Lagrangean-based decomposition techniques for solving the temporal decomposition of a continuous flexible process network. They use subgradient methods to solve the decomposed problem and find that temporal decomposition strategies result in a reduction of computational time of several orders of magnitude.

From the papers mentioned above, it is evident that temporal decomposition is an efficient solution approach for multi-period planning problems. It has been found to provide a tighter bound on the optimal solution and to have better convergence properties than Lagrangean spatial decomposition in multi-site problems. However, there is no rigorous proof and generalization of the observed result. One objective of this paper is to compare the bounds obtained through Lagrangean temporal and spatial decompositions for a class of MILP problems derived from the lot-sizing problem with setup and sequence-dependent changeover times (Pochet & Wolsey, 2006). The second objective is to use the eco-

* Corresponding author. Tel.: 1 412 268 3642; fax: +1 412 268 7139.
E-mail address: grossmann@cmu.edu (I.E. Grossmann).

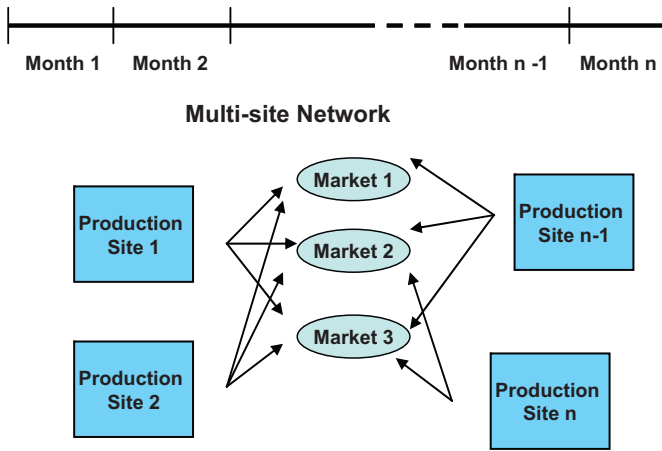


Fig. 1. Network of production sites and markets for a multi-period planning problem.

conomic interpretation of the Lagrange multipliers to provide a reduced dual search space and accelerate the convergence of the optimal multipliers.

This paper is organized as follows. Section 2 presents the details of the MILP formulation for the production planning problem. Section 3 describes the implementation of temporal and spatial decomposition of the MILP problem in Section 2. In Section 4 we review some important theoretical concepts and in Section 5 we introduce a result where the dual gap of temporal decomposition is found to be at least as small as the dual gap for spatial decomposition. Sections 6 and 7 contain our novel approach for exploiting the economic interpretation of the Lagrange multipliers to reduce the search space for the optimal multipliers. Section 8 presents four numerical examples of increasing size and complexity for the multi-site multi-period planning problem where the theoretical result is confirmed, and where the economic interpretation of the multipliers is used to speed the convergence of numerical algorithms for solving the decomposed problem. Finally, Section 9 presents our conclusions and ideas for future work.

2. Problem statement

Given is a set of products that are manufactured in several continuous multi-product production sites and shipped to a set of markets where they are sold. Let I , S , and M be the sets of products, production sites, and markets, respectively. Fig. 1 shows the multi-period, multi-site network structure.

There is a finite time horizon divided into time periods of length L_t . The set of time periods is denoted by T . Given is also a forecast of the demand of each product in each market at the end of the time periods. The problem is to determine the production in each site, the inventory levels, and the amounts of products shipped to each market during each time period in order to maximize the profit. We assume that the size of the problem may prohibit its direct solution, and we consider that temporal and spatial Lagrangean decomposition techniques are alternatives to overcome this challenge. One objective of this work is to rigorously compare the strength of the Lagrangean duals resulting from each decomposition scheme. Another objective is to illustrate how the economic interpretation of the Lagrange multipliers of the constraints that are relaxed in both decompositions can be used to reduce the dual search space.

The following mixed-integer linear programming (MILP) model, which is formulated in a generic way, corresponds to the profit

maximization planning problem described above.

$$\max_{s,t} \pi = \sum_{t \in T} \sum_{s \in S} \sum_{m \in M} \beta_t^m s_t^{s,m} - \sum_{t \in T} \sum_{s \in S} (\alpha_t^s x_t^s + \delta_t^s v_t^s + TC_t^s trn_t^s) - \sum_{t \in T} \sum_{s \in S} \sum_{m \in M} \gamma_t^{s,m} f_t^{s,m} \tag{1.1}$$

$$v_{t-1}^s + x_t^s = \sum_{m \in M} f_t^{s,m} + v_t^s \quad \forall t \in T, s \in S \tag{1.2}$$

$$x_t^s \leq x_s^{UB} stp_t^s \quad \forall t \in T, s \in S \tag{1.3}$$

$$a_t^s x_t^s + bs_t^s stp_t^s + bt_t^s trn_t^s \leq L_t \quad \forall t \in T, s \in S \tag{1.4}$$

$$Cs_t^s(stp_t^s) + Ct_t^s(trn_t^s) + Ct_t^s(trn_{t+1}^s) \leq e_t^s \quad \forall t \in T, s \in S \tag{1.5}$$

$$f_t^{s,m} = s_t^{s,m} \quad \forall t \in T, s \in S, m \in M \tag{1.6}$$

$$\sum_{s \in S} s_t^{s,m} \leq d_t^m \quad \forall t \in T, m \in M \tag{1.7}$$

$$x_t^s \leq x_s^{UB}, \quad v_t^s \leq v_s^{UB} \quad \forall t \in T, s \in S \tag{1.8}$$

$$f_t^{s,m} \leq f_{s,m}^{UB} \quad \forall t \in T, s \in S, m \in M \tag{1.9}$$

$$x_t^s \in \mathbb{N}_+^{|I|}, \quad v_t^s \in \mathbb{N}_+^{|I|}, \quad trn_t^s \in \mathbb{N}_+^{|I| \times |I| \times \# \text{ trn variables}} \tag{1.10}$$

$$f_t^{s,m} \in \mathbb{N}_+^{|I|}, \quad s_t^{s,m} \in \mathbb{N}_+^{|I|} \quad \forall t \in T, s \in S, m \in M \tag{1.11}$$

$$stp_t^s \in \{0, 1\}^{|I|} \quad \forall t \in T, s \in S \tag{1.12}$$

$$0 \leq trn_t^s \leq 1 \quad \forall t \in T, s \in S \tag{1.13}$$

Eq. (1.1) represents the maximization of profit, computed as sales minus production, inventory, transition, and transportation costs. The coefficients β_t^m , α_t^s , δ_t^s , TC_t^s , and $\gamma_t^{s,m}$ are row vectors of length $|I|$, where I is the set of products produced and sold in the multi-site network. Eq. (1.2) is the production site mass balance; x_t^s represents production, v_t^s inventory levels, and $f_t^{s,m}$ amount of product shipped from s to m . Constraint (1.3) enforces the condition that a product can only be produced if there is a setup assigned to it (stp_t^s). Constraint (1.4) enforces the time balance at each production site involving the transition variables trn_t^s . The row vector a_t^s contains the inverse of the production rates of all products, while bs_t^s and bt_t^s are vectors with set up and transition times. L_t is the duration of time period t . Constraint (1.5) is a compact representation of a set of general sequencing constraints. In general, assume there are $k = 1, \dots, K$ constraints in this set. Then Cs_t^s is a matrix with K rows and $|I|$ columns, and Ct_t^s and Ct_t^s are matrices with the same number of rows and $|I| \times |I| \times (\# \text{ trn variables})$ columns. The expression $\# \text{ trn variables}$ represents the different types of transition variables used in the model, for instance, transition within time periods, transitions across time periods, etc. The vector of right hand side coefficients e_t^s has dimension K . Eq. (1.6) expresses the condition that all products that arrive at a market are sold. The vector d_t^m contains the market demands for all the products at each time period. The rest of the constraints involve the upper bounds, ranges, and integrality conditions of the decision variables. It is important to note that the transition variables trn_t^s are continuous and bounded between zero and one. We assume that constraint (1.5) contains sequencing constraints of the type shown in Appendix A and proposed by Erdirik-Dogan and Grossmann (2008), where transition variables always take values of 0 or 1, even if they are declared as continuous.

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