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Portfolio optimization in the presence of dependent financial returns with long memory: A copula based approach

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ABSTRACT

In this paper, we seek to examine the effect of the presence of long memory on the dependence structure between financial returns and on portfolio optimization. First, we focus on the dependence structure using copulas. To select the best copula, in addition to the goodness of fit tests, we employ a graphical method based on visual comparison of the fitted copula density and the smoothed copula density estimated by wavelets. Moreover, we check the stability of the copula parameter. The empirical results show that the long memory affects the dependence structure. Second, we analyze the impact of this dependence structure on the optimal portfolio. We propose a new approach based on minimizing the Conditional Value at Risk and assuming that the dependence structure is modeled by the copula parameter. The empirical results show that our approach outperforms the traditional minimizing variance approach, where the dependence structure is represented by the linear correlation coefficient.

1. Introduction

Modern Portfolio Theory argues that investors can benefit from diversification by investing in financial assets with lower correlations. It assumes that the financial returns are subject to joint-normal distribution. Therefore, the dependence between financial returns is fully described by the linear correlation coefficient and efficient portfolios are given by the traditional mean variance optimization program (Markowitz, 1952).

However, empirical studies find that the assumption of normality of financial returns distribution is not hold. The departures from normality have been examined using the skewness (the third moment of distribution) and the kurtosis (the fourth moment of distribution). Some studies find evidence of negative skewness and excess kurtosis. Specifically, negative skewness indicates a higher probability of negative returns, that is, the market gives higher probability to decreases than increases in asset pricing while excess kurtosis makes extreme observations more likely than in the normal case, which means that the market gives higher probability to extreme observations than in normal distribution. Others studies find evidence of time-varying skewness (Harvey and Siddique, 1999) and time-varying kurtosis (Jondeau and Rockinger, 2003; Brooks et al., 2005). Moreover, there is evidence of long memory in the mean and in the variance of financial returns (Crato, 1994; Lobato and Savin, 1998; Sadique and Silvapulle, 2001). Consequently, the linear correlation coefficient is no longer suitable and can largely lead to misleading results (for more discussions about the bias in the linear correlation coefficient, see Embrechts et al. (1999, 2002) and Forbes and Rigobon (2000)). Furthermore, several authors find evidence of asymmetry in the dependence between stock market returns (Longin and Solnik, 2001; Ang and Bekaert, 2002; Ang and Chen, 2002; Das and Uppal, 2004; Patton, 2004; Hong et al., 2007) and between exchange rate returns (Beine, 2004; Li, 2011). In particular, stock market returns exhibit greater dependence during market downturns than market upturns and exchange rate returns seem to be more correlated in periods of depreciation than in periods of appreciation which increase the downside risk.

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To reproduce the asymmetric dependence between financial returns, some authors propose to rely on the concept of exceedance correlation[1] (Ang and Chen, 2002) and the extreme value theory[2] (Longin and Solnik, 2001; Hartmann et al., 2004; Poon et al., 2004; Beine et al., 2010). An important limitation of these methods, is that they are subject to the problem of choosing the appropriate threshold and the use of the extreme value distribution such as Pareto distribution.

Others authors consider multivariate Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model[3] with skewness (Harvey and Siddique, 1999) and/or kurtosis (Jondeau and Rockinger, 2003; Brooks et al., 2005). Few authors use regime-switching models (Ang and Bekaert, 2002; Ang and Chen, 2002). Although these models allow for time-varying conditional correlations, they cannot reproduce asymmetries in asymptotic tail dependence. Tail dependence refers to dependence that arises between random variables from extreme observations.

To overcome this shortcoming, we propose an alternative approach to modeling the dependence structure of multivariate data, without imposing any assumption in marginal distributions, based on copula theory. The advantage of copula lies in separating marginal distributions and dependence structure from joint distribution. In particular, we consider Archimedean copulas since they allow for asymmetry and dependence in the extreme tails.

Archimedean copula functions have been applied by several authors to model the dependence structure between stock market returns (Costinot et al., 2000; Patton, 2004; Canela and Collazo, 2006; Kole et al., 2005; Jondeau and Rockinger, 2006; Hu, 2006; Rodriguez, 2007; de Melo Mendes and Kolev, 2008; Sun et al., 2009; Ignatievay and Platen, 2010; Aloui et al., 2011; Chollete et al., 2011; Garcia and Tsafack, 2011) and the dependence structure between exchange rate returns (Liu, 2006; Manner, 2007).

A major implication of this asymmetric dependence between financial returns is that traditional mean variance optimization analysis portfolios are not efficient with respect to their effective risk profile. To take into account the asymmetric dependence structure between financial returns when determining the optimal portfolio, Fantazzini (2004) proposes an optimization program for a portfolio of stock market indexes, where the dependence structure is modeled by Kendall's tau instead of the linear correlation coefficient. DiClemente and Romano (2004) propose an optimization program for a portfolio of credit risks, where the dependence structure between credit assets is modeled by copula functions. He and Gong (2009) propose also an optimization program for a portfolio of market and credit risks, where their dependence structure is modeled by copula functions.

The objective of this paper is twofold. First, we focus on modeling the dependence structure between financial returns using copula functions. As financial returns, we consider one pair of daily stock market returns and one pair of daily exchange rate returns. In particular, we investigate whether the presence of long memory behavior affects the dependence structure between financial returns. Second, we study the influence of the dependence structure between financial returns on the optimal portfolio.

To select the best copula, the authors compare the values of the log-likelihood, the information criterion and the distance of Cramér-von Mises between the fitted copula and the empirical copula. In this paper, we apply, in addition, an innovative graphical method based on visual comparison of the fitted density copula and the smoothed density copula. More precisely, the smoothed density copula is estimated by using a non-parametric estimator based on the wavelet decomposition as advanced by Genest et al. (2009b). Further, we check for stability of the copula parameter.

To determine the optimal portfolio, we refer first to the classical mean risk framework proposed by Markowitz (1952). We consider that risk is measured by variance and that the dependence structure between financial returns is modeled by the linear correlation coefficient. Thus, the optimization problem involves on minimizing the variance for a given return. Then, we replace the linear correlation coefficient by the copula parameter to reproduce the true dependence structure between financial returns.

It is well known that variance has been criticized because it is a symmetric measure and treats downside risk and upside risk in the same way while the investors assign greater importance to downside risk than to upside risk (Post and van Vliet, 2004; Ang et al., 2006). In the same context, Tsafack (2009) shows that the use of symmetric dependence tends to underestimate extreme risk in the presence of asymmetric dependence. For the accuracy of risk measure, some authors introduced several downside risk measures in their optimization program. As downside risk measures, Markowitz (1959) considers the semi-variance. Using safety first principle,[4] Roy (1952) and Arzac and Bawa (1977) consider Value at Risk (VaR).[5] To take into account the tail events, Susmel (2001) uses extreme value theory to reproduce the fact that the returns are fat-tailed. In this paper, we provide an alternative method for modeling the tail events based on copulas. In addition, VaR has been criticized for not being a coherent risk measure. Indeed, VaR is not sub-additive, i.e., the risk associated with a given portfolio can be larger than the sum of the stand alone risks of its components when measured by the VaR (for more details, see Artzner et al. (1999), McNeil et al. (2005), and Gilboa (2009)). For this reason, we retain the Conditional Value at Risk (CVaR)[6] as downside risk measure instead of the VaR. Following DiClemente and Romano (2004) and He and Gong (2009), we develop an optimization program based on minimizing the CVaR and, where the dependence structure between financial returns is modeled by the copula parameter.

Our findings have significant implications. First, understanding and measuring the interdependence between financial returns is important for international financial market integration and portfolio diversification. Second, the optimization program proposed here can help the investors to compute more accurately the extreme risk of the portfolio.

The remainder of this paper is organized as follows. In the next section, we review briefly the main concepts of the copula theory and we present an empirical application to modeling the dependence structure between financial returns. In Section 3, we analyse the effect of this dependence structure on the optimization portfolio. In Section 4, we conclude the paper.

2. Modeling the dependence structure between financial returns using copula functions

In this section, we propose to model the dependence structure between financial returns using copula functions. Firstly, we

[1] The exceedance correlation between two series is defined as the correlation for a subsample in which the series are simultaneously lower or greater than the corresponding thresholds.

[2] The extreme value theory is used to characterize the distribution of a variable conditionally to the fact that its values are beyond a certain threshold and the asymptotic distribution is obtained when this threshold tends to infinity.

[3] In particular, they consider GARCH-CCC model of Bollerslev (1990) and GARCH-DCC model of Engle (2002).

[4] The safety first criterion involves creating a portfolio based on a minimum level of portfolio return, which is called the minimum acceptable return.

[5] $VaR_\alpha(X)$ at confidence level α % is defined as the maximum expected loss on an investment: $VaR_\alpha(X) = \sup\{x \setminus P[X \geqslant x] > \alpha\}$.

[6] $CVaR_\alpha(X)$ at the confidence level α % is defined as the conditional expected loss under the condition that it exceeds the Value at Risk (VaR): $CVaR_\alpha(X) = E[X \setminus X \geqslant VaR_\alpha(X)]$. For more discussions on the use of CVaR in optimization problems, see Uryasev (2000), Rockafellar and Uryasev (2000, 2002), Pflug (2000), Krokhmal et al. (2002), Gaivoronski and Pflug (2005), and Quaranta and Zaffaroni (2008).