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Calculation of the power factor using the neutron diffusion hybrid equation

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ABSTRACT

In this paper, we used a neutron diffusion hybrid equation with an external neutron source to calculate nuclear power factors in each fuel element in the reactor core. We used the nodal expansion method to obtain the neutron flux for a given control rods bank position. The results were compared with results obtained for eigenvalue problem near criticality condition and fixed source problem during the start-up of the reactor, where external neutron sources are extremely important for the stabilization of external neutron detectors.

1. Introduction

The objective of coarse mesh nodal methods such as, Nodal Expansion Method (NEM) (Finnemann et al., 1977), Semi-Analytic Nodal Method (SANM) (Yeong-Il et al., 1999), Analytic Nodal Method (ANM) (Greenman et al., 1979) and Flux Expansion Method (FEM) (Langenbuch et al., 1977) is to accurately obtain the eigenvalue and the neutron flux in order to determine power distribution in the nuclear reactor core. Some reference methods were developed to validate those new nodal methods, such as the IAEA benchmark (Wagner et al., 1977). This benchmark was obtained solving the neutron diffusion equation defined as an eigenvalue problem, in 3D geometry, for two energy groups which supplies the eigenvalue and the power distribution in each fuel element as a comparison result.

The objective of this paper is to calculate the power distribution for different subcritical states as a function of the position of the control rods banks withdrawn of the reactor core, using the neutron diffusion hybrid equation model (Da Silva et al., 2011), from the reactor start-up, that is, with the reactor core in such a subcritical state that the presence of external sources is fundamental for the stabilization of external neutron detectors in the reactor core. And near the criticality condition, where it is assumed that neutron production by the external sources is negligible when compared with neutron production by fissions in the reactor core.

2. Neutron diffusion hybrid equation

The coarse mesh nodal methods, such as NEM, divide the volume of the reactor core into a set of contiguous parallelepipeds, named nodes, in which the nuclear multigroup parameters are uniform. One of the main characteristics of this method is that it uses the transverse-integrated diffusion equation, generating a set of three one-dimensional equations coupled through transverse leakage terms, the solution of which supplies a relation between average fluxes and average net currents on the faces of the node. The starting point of this method is the modified neutron continuity equation, which integrated in volume $V_n = a_x^n a_y^n a_z^n$ of node n, for two energy groups is given by:

$$\sum_{u=x,y,z} \frac{1}{a_u^n}\left[\bar{J}_{gur}^n - \bar{J}_{gul}^n\right] + \sum_{rg}^{n} \overline{\phi}_g^n = \sum_{\substack{g'=1 \\ g'\neq g}}^{2} \sum_{gg'}^{n} \overline{\phi}_{g'}^n + \alpha\chi_g \sum_{g'=1}^{2} \nu \sum_{fg'}^{n} \overline{\phi}_{g'}^n + \beta\bar{S}_g^n, \quad (1)$$

where the terms are defined as:

$\bar{J}_{gus}^n$ is the neutron net current of group g on face s = l, r, in direction u; Σ_{rg}^n is the macroscopic removal cross section of group g; $\Sigma_{gg'}^n$ is the macroscopic scattering cross section from group g' to group g; χ_g is the fission spectrum of group g; $\nu\Sigma_{fg'}^n$ is the product of the average number of neutrons emitted by fission by the macroscopic fission cross section of group g'; $\bar{S}_g^n$ is the external neutron source of group g; $\overline{\phi}_g^n$ is the neutron flux of group g; a_u^n is the size of node n in direction u; α, β is the hybrid parameters of the equation.

Parameters α and β are defined as:

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$$\beta \equiv -\rho, \tag{2}$$

and

$$\alpha \equiv 1 - \frac{\beta^2(1-\beta)}{10}, \tag{3}$$

where ρ is the calculated reactivity of the reactor core. The parameters α and β will be used to adjust Eq. (1) according to the subcriticality level of the reactor core. During the reactor start-up with control rods banks fully inserted in the core, the values of the parameters are such that $\beta = 1$ and $\alpha = 1$. That means, we will have the continuity equation defined for a fixed source problem. Solving this equation, i.e., after flux convergence, we can obtain the reactivity ρ (Da Silva et al., 2011). Near the criticality, with control rods banks withdrawn of the reactor core, the reactivity is null, therefore $\beta = 0$ and $\alpha = 1$. In this case we will have the neutron continuity equation defined for the criticality problem. On the condition that the control rods banks are partially inserted in the core, the reactivity ρ is calculated according to the procedure described in reference (Da Silva et al., 2011).

The advantage of using a diffusion hybrid equation is to represent the contribution of the neutron source during the reactor start-up, from which the eigenvalue problem does not consider the contribution of the external source term. Using this equation the transition from a fixed source problem (during start-up of the reactor) to an eigenvalue problem (near criticality) is possible.

During this research it was found out that the behavior of parameters α and β significantly affect the neutron flux, as well as the nuclear power density. Therefore, it was necessary to analyze the behavior of these parameters. In this analysis we found out that the parameter α should change very little close to the unit. And close to criticality, parameter β goes to zero. For this reason we changed the definitions of these parameters in the neutron diffusion equation, as compared to reference (Da Silva et al., 2011).

The neutron flux can be obtained from Eq. (1) for a given intensity of the external neutron source and using the average multigroup nuclear parameters in the nodes.

3. Power factor calculations

One of the main objectives of general calculations for nuclear reactors is to determine the power distribution in the reactor core, for each fuel element. With average neutron flux obtained by Eq. (1), we can calculate the average power density at the node using the following relation:

$$\overline{p}^n = \sum_{g=1}^{2} \gamma_g \sum_{fg}^{n} \overline{\phi}_g^n, \tag{4}$$

where γ_g is the energy produced per fission for energy group g. The average power density of the reactor core can be determined from the average power density on the node,

$$\overline{P}^{Core} = \frac{1}{V_{Core}} \sum_{n=1}^{N} \overline{p}^n V_n, \tag{5}$$

where N is the total number of nodes and V_{Core} the total volume of the reactor core.

In order to validate the results obtained, we will calculate the power factor f_{Power}^{FE} for each fuel element (FE). Initially, we will determine the average power density in the fuel element $\overline{P}^{FE}$, defined as the sum of the products of the average power density by the volume of the node, for all the nodes of the fuel element, divided by the volume of the fuel element as follows:

$$\overline{P}^{FE} = \frac{1}{V_{FE}} \sum_{n \in FE} \overline{p}^n V_n = \frac{1}{\sum_{n \in FE} V_n f^n} \sum_{n \in FE} \overline{p}^n V_n f^n = \frac{1}{\sum_{n \in FE} V_n} \sum_{n \in FE} \overline{p}^n V_n, \tag{6}$$

where V_{FE} is the total volume of the fuel element, V_n the volume of the node and f^n the contribution factor of the volume of the node (in the case of 1/4 core symmetry), as illustrated in Fig. 1. It is important to emphasize that in the case of an entire core $f^n = 1$ for any node.

Note that for core power calculation, we can define a contribution factor of the fuel element f^{FE} for the core, in such a way that:

$$\sum_{FE} V_{FE} f^{FE} = V_{Core}, \tag{7}$$

and thus, Eq. (5) can be written as a function of the average power of the fuel element as follows:

$$\overline{P}^{Core} = \frac{1}{\sum_{FE} V_{FE} f^{FE}} \sum_{FE} \overline{P}^{FE} V_{FE} f^{FE}, \tag{8}$$

where f^{FE} is the contribution factor of the fuel element to the volume of the reactor core, as illustrated in Fig. 2.

The power factor f_{Power}^{FE} is defined as the ratio between the average power of the fuel element and the average power of the core, given by the following equation:

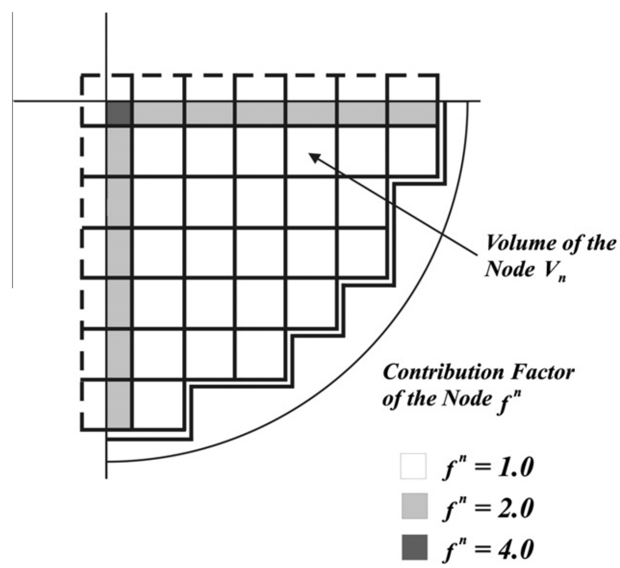

Fig. 1. Contribution factor of the node to the power of the reactor core.

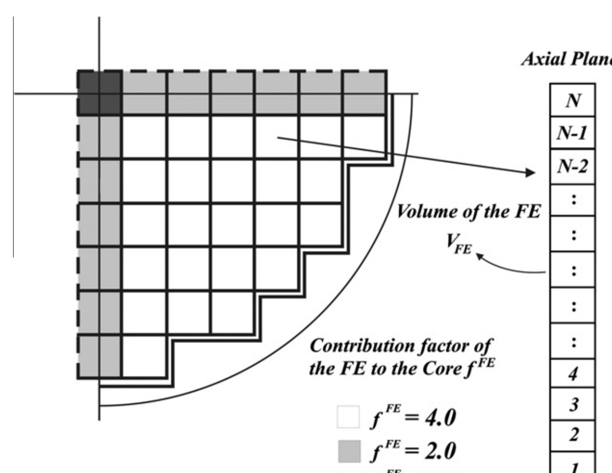

Fig. 2. Contribution factor of the fuel element to the core.