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A robust optimization approach to asset-liability management under time-varying investment opportunities

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ABSTRACT

This paper presents an asset liability management model based on robust optimization techniques. The model explicitly takes into consideration the time-varying aspect of investment opportunities. The emphasis of the proposed approach is on computational tractability and practical appeal. Computational studies with real market data study the performance of robust-optimization-based strategies, and compare it to the performance of the classical stochastic programming approach.

1. Introduction

Asset-liability management (ALM) is one of the classical problems in financial risk management. Typically, ALM involves the management of assets in such a way as to earn adequate returns while maintaining a comfortable surplus of assets over existing and future liabilities. This problem is faced by a number of financial services companies, such as pension funds and insurance companies. As we will explain in more detail later, the problem of finding optimal ALM policies is computationally challenging, and many of the approaches for implementation described in the literature are too computationally intensive to implement in practice. In this paper, we propose and study the performance of a robust-optimization-based approach for handling the classical ALM problem. Our focus is on computational tractability and practical implementation.

Analytical solutions for optimal dynamic investment strategies of the ALM type are available for some limited cases (see, for example, the classical papers of Samuelson, 1969; Merton, 1969; or, more recently, Kim and Omberg, 1996; Wachter, 2002). However, mostly numerical methods are used in practice. These numerical approaches fall into three broad categories. The first is dynamic programming—the state space is discretized and the optimal allocation strategy is found by backward induction (see, for example, Barberis, 2000; Detemple and Rindisbacher, 2008). The second category is simulation-based approaches (see, for example, Brandt et al., 2005; Boender, 1997). The third category, prevalent in the operations research and practitioner literature, is stochastic programming techniques (see Ferstl and Weissensteiner, 2011; Consiglio et al., 2006; Boender et al., 2005; Kouwenberg, 2001; Ziemba and Mulvey, 1998, among others). The latter techniques usually focus on finding optimal investment rules over a set of scenarios for the future returns on the assets and the liabilities of the company.

While such methods have been successfully applied in some instances (Gondzio and Kouwenberg, 2001; Consigli and Dempster, 1998; Consiglio et al., 2008; Escudero et al., 2009), they are still difficult to use in practice for several reasons. First, ALM is inherently a multiperiod problem, and the number of scenarios needed to represent reality satisfactorily increases exponentially with the number of time periods under consideration. Thus, the dimension of the optimization problem, and correspondingly its computational difficulty, increases. Many of the papers that suggest scenario-based approaches for ALM adopt approximations to the state space or relaxations of the optimization problem to make the problem manageable in practice (see, for example, Bogentoft et al., 2001). Second, the scenario generation itself requires sophisticated statistical techniques, which is a deterrent to practitioners who need to make decisions in a short amount of time. Finally, often little is known about the specific distributions of future uncertainties in the ALM problem, and little data are available for estimating the probability distributions of these uncertainties. In many cases, it

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may be preferable to provide general information about the uncertainties, such as means, ranges, and deviations, rather than generating specific scenarios.

This paper adopts a numerical approach, robust optimization, that can be classified in its own category, but has overlap with the dynamic programming and stochastic programming approaches. Specifically, robust optimization can be used to address the same type of problems as dynamic programming and stochastic programming do; however, it takes a worst-case approach to optimization formulations. (For detailed discussion of the relationships among the three numerical methods, see chapter 10 in Fabozzi et al., 2007.) This is not as restrictive as it sounds at first. The robust optimization approach solves an optimization problem assuming that the uncertain input data belong to an uncertainty set, and finds the optimal solution if the uncertainties take their worst-case values within that uncertainty set. As we will explain in more detail in Section 3, the shape and the size of the uncertainty set can be used to vary the degree of conservativeness of the solution and to represent an investor's risk preferences.

In industry, robust optimization has been used only in asset management, and primarily to incorporate the uncertainty introduced by estimation errors into the mean–variance portfolio allocation framework. For example, Goldfarb and Iyengar (2003) consider robust mean–variance portfolio allocation strategies under various ellipsoidal and interval uncertainty sets for the input parameters (means and covariance matrices) derived from regression analysis. Ceria and Stubbs (2006) introduce the zero-net alpha-adjustment robust framework to reduce the conservativeness of robust mean–variance strategies under ellipsoidal uncertainty sets for the input parameters. Robust investment strategies in a multiperiod setting have been studied by Ben-Tal et al. (2000) and Bertsimas and Pachamanova, 2008.

Given the fact that ALM is concerned with ensuring a level of minimum guaranteed performance to meet future liabilities, robust-optimization-based strategies that place special emphasis on the worst-case realizations of uncertainties are particularly appealing in the ALM context.

We propose a tractable robust approach to ALM for pension funds. Our contributions can be briefly summarized as follows. First, we derive the robust counterpart of the ALM problem when future uncertainties are represented by ellipsoidal sets. These uncertainty sets can be naturally generated from statistical factor models for the uncertain variables in the problem. Second, we model the time-varying aspect of asset returns and interest rates by presenting a case study of the robust counterpart when asset returns and interest rates follow a vector-autoregressive (VAR) process. Finally, we design numerical experiments to study the performance of the robust ALM model, and benchmark it against the performance of another ALM strategy used in practice, a stochastic programming formulation. (We are primarily concerned with benchmarking our approach against traditional stochastic programming approaches since stochastic programming approaches are most widely used in practice.)

The rest of this paper is organized as follows. In Section 2, we present the ALM model for pension funds. A brief primer on robust optimization is given in Section 3, and a general robust formulation of the ALM model for pension funds is derived in Section 4. Section 5 presents an example of the robust formulation under specific assumptions on the dynamics of asset returns and interest rates. Computational experiments with real market data are presented in Section 6. Section 7 summarizes our findings.

Notation: In the paper, we use tilde ($\tilde{*}$) to denote randomness; e.g., $\tilde{z}$ denotes random variable z. Boldface is used to denote vectors; boldface and capital letters are used to denote matrices. For example, $\boldsymbol{a}$ is a vector and $\boldsymbol{A}$ is a matrix.

2. ALM model for pension funds

A typical pension fund collects premiums from sponsors or currently active employees, pays pensions to retired employees, and also invests available funds. The fund manages assets so that at each time period the total value of all assets exceeds the company's future liabilities. At the same time, the fund minimizes the contribution rate by the sponsor and active employees of the fund (see, for example, Bogentoft et al., 2001). Therefore, the ALM problem for a pension fund is to determine an optimal contribution rate and investment strategy during an investment horizon of length T.

We assume that a portfolio is constructed from M risky assets and investment decisions are made at discrete time $t = 0, \ldots, T$, where $t = 0$ represents today. Securities are denoted by $m = 1, 2, \ldots, M$, and $m = 0$ identifies the risk-free asset.

Depending on the state of the pension fund at a particular time t, the fund manager makes a decision on the value of contributions to the fund and the portfolio allocation. Let h_t^m, s_t^m and b_t^m denote the amount of asset m to be held, sold and bought at time t, respectively. Exogenous and endogenous factors cause uncertainty that needs to be incorporated into the decision making process. Asset returns $\tilde{r}_t^m$ for $m = 1, \ldots, M$, as well as the risk-free returns $\tilde{r}_t^0$, are random variables. While the liabilities to be paid out at each stage l_t are known at time 0, the total present value at time t of all future liabilities, $\tilde{L}_t$, is unknown because changes in the discount rates over time affect the present value of the cash flows. The amount of wages W_t at t is considered certain. Further, γ_t denotes the contribution as a percentage of wages at time t. See Table 1 for a summary of notations. The optimization model constraints are defined as follows.

Balance constraints: At time t, a balance constraint determines the wealth gained from each asset m. The latter consists of holdings and gains from trading at the previous time period $t - 1$. The holding in each asset m at time t equals the amount received at time $t - 1$ plus the return earned in one time period.

$$h_t^m = \left(1 + \tilde{r}_{t-1}^m\right) \cdot h_{t-1}^m - s_t^m + b_t^m, \quad t = 1, \ldots, T, \; m = 1, \ldots, M \qquad (1)$$

Amount of cash: The amount of cash at t is equal to the value of the amount invested at $t - 1$ plus cash resulting from changes in positions or wage contributions minus the liabilities to be paid out at time t. This is expressed as follows:

$$h_t^0 = (1 + \tilde{r}_{t-1}^0) \cdot h_{t-1}^0 + \sum_{m=1}^{M}(1 - c_s)s_t^m - \sum_{m=1}^{M}(1 + c_b)b_t^m + \gamma_t W_t - l_t, \quad t = 1, \ldots, T \qquad (2)$$

Table 1
Description of notation.

Notation	Description
Parameters	
T	Investment horizon
M	Number of investment assets
ψ	Target funding (asset/liability) ratio
c_b, c_s	Transaction costs for buying and selling, respectively
l_t	Amount (liabilities) paid out to retirees at time t
W_t	Amount of wages at time t
Decision variables	
h_t^m	Holding in asset m at time t
s_t^m	Amount sold of asset m at time t
b_t^m	Amount bought of asset m at time t
γ_t	Contribution as percentage of wages at time t
Random variables	
$\tilde{r}_t^m$	Return on asset m between time t and $t + 1$
$\tilde{L}_t$	Present value of the total amount of future outstanding liabilities at time t