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A multi-period portfolio selection optimization model by using interval analysis

Yong-Jun Liu [a], Wei-Guo Zhang [a,*], Pu Zhang [b]

[a] School of Business Administration, South China University of Technology, Guangzhou, 510641, PR China
[b] School of Economics & Management, Changzhou University, Changzhou, 213164, PR China

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ABSTRACT

Due to few historical data that can be obtained in an emerging securities market, the future returns, risk and liquidity of securities cannot be forecasted precisely. The investment environment is usually fuzzy and uncertain. To handle these imprecise data, this paper discusses a fuzzy multi-period portfolio optimization problem where the returns, risk, and liquidity of securities are represented by interval variables. By taking the return, risk, liquidity and diversification degree of portfolio into consideration, an interval multi-period portfolio selection optimization model is proposed with the objective of maximizing the terminal wealth under the constraints of the return, risk and diversification degree of portfolio at each period. In the proposed model, a proportion entropy is employed to measure the diversification degree of portfolio. Using the fuzzy decision-making theory and multi-objective programming approach, the proposed model is transformed into a crisp nonlinear programming. Then, we design an improved particle swarm optimization algorithm for solution. Finally, a numerical example is given to illustrate the application of our model and demonstrate the effectiveness of the designed algorithm.

1. Introduction

Portfolio selection problem is one of the most hot topics in finance. The original research on portfolio selection dates back to Markowitz (1952) on mean–variance model for single period portfolio selection problem. It has played an important role in the development of modern portfolio analysis. After Markowitz's pioneer work, numerous scholars extended the classical mean–variance model into multi-period portfolio selection, see for example Mossion (1968), Dumas and Lucinao (1991) and Pliska (1997). However, studies on multi-period portfolio selection have been dominated by using the probability theory to deal with the uncertainty of financial market. Though probability theory is one of the main tools used for handling uncertainty in finance, it cannot describe uncertainty completely since there are many non-probabilistic factors affect the financial markets.

In real world, investors may face with imperfect information data and hence must deal with uncertain, imprecise and vague data. So, in many cases, these imprecise data or vague information are represented by natural language and subjective statement. Following the widely used fuzzy set theory in Zadeh (1965), researchers have realized that they could use the fuzzy set theory to investigate portfolio selection problems under uncertain environment. Numerous models have been proposed by using different approaches. Ramaswamy (1998) presented a bond portfolio selection model using fuzzy decision theory. Carlsson et al. (2002) considered portfolio selection problems under possibility distributions and presented an algorithm for finding an exact optimal solution to these problems. Fang et al. (2006) proposed a linear programming model for portfolio rebalancing with transaction costs based on fuzzy set theory. Zhang et al. (2007, 2009, 2010) proposed the portfolio selection models based on possibilistic means and possibilistic variances of fuzzy numbers. Li et al. (2010) proposed several fuzzy mean–variance–skewness models in fuzzy environment with fuzzy returns. By using experts' judgments, Tanaka et al. (2000) constructed two kinds of portfolio optimization models based on fuzzy probabilities and possibility distributions. Recently, some researchers investigated fuzzy multi-period portfolio selection problem. Sadjadi et al. (2011) formulated a fuzzy multi-period portfolio selection model with different rates for borrowing and lending by using fuzzy set theory. Zhang et al. (2012) presented a mean–semivariance–entropy model for multi-period portfolio selection based on possibility theory. Liu et al. (2012) investigated multi-period portfolio selection problem in fuzzy environment by using multiple criteria.

Notice that all previous fuzzy portfolio selection literatures use fuzzy set theory to handle the imprecise input data of financial markets. In these models, it is often assumed that the possibility distribution functions of returns are known. However, in real world, it is not always easy for an investor to determine them as pointed out by Lai et al. (2002). In the decision-making literature, interval analysis method is a more useful and simple approach for a decision-maker to handle this kind of uncertainty. This approach assumes that the data of a decision-making problem are not crisp numbers but interval numbers.

* Corresponding author. Tel.: +86 208711421; fax: +86 208711405.
E-mail addresses: liuyongjun0202@yahoo.com.cn (Y.-J. Liu), wgzhang@scut.edu.cn (W.-G. Zhang).

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Recently, a few researches have surveyed fuzzy portfolio selection problem by using interval analysis approach. Moore (1966) introduced a series of interval analysis approaches. These approaches assume that the data of a decision-making problem are not single values but may vary in given ranges. On this subject, Alefeld and Mayer (2000) presented both theory and some applications of interval analysis. Following the widely used interval programming approaches, such as Inuiguchi and Sakawa (1995), Chinneck and Ramadan (2000) and Jiang et al. (2008), some applications of them can be found in dealing with portfolio selection problems. Parra et al. (2001) presented a goal programming model for portfolio selection problem by taking three criteria into consideration including return, risk and liquidity. These objectives and targets were characterized by interval variables. Lai et al. (2002) proposed an interval programming portfolio selection model by quantifying the expected return and the covariance as intervals. Ida (2003, 2004) investigated multi-objective portfolio selection problem with interval coefficients. Giove et al. (2006) considered a portfolio selection problem in which the prices of the securities were treated as interval variables. Bhattacharyya et al. (2011) extended the classical mean–variance portfolio selection model into mean–variance–skewness model with consideration of transaction cost by using interval analysis. In Inuiguchi and Tanino (2000), the minimax regret criterion had been applied to formulate possibilistic portfolio selection problems under the assumption that the return rates of assets were given and wanted to minimize the worst regret. Mitchell (2001) explored further the possible effects of the decision interval upon the risky asset's share in the portfolio, when there may be positive or negative serial correlation of risky returns. Meanwhile, he also considered both the case in which rebalancing was constrained to maintain a constant risky share at the start of each time period within the decision interval and the case in which the risky share must follow a precommitted but not necessarily constant sequence. Li and Xu (2007) dealt with a possibilistic portfolio selection model with interval center values. Liu (2011) discussed the uncertain portfolio selection problem where the returns of assets were represented by interval data.

Though all previous interval programming portfolio optimization literatures show that interval analysis has demonstrated considerable success in handling the uncertainty of financial markets. However, they only consider single-period portfolio decision-making problems. It is well known that, in practical investment, investors' behaviors are usually multi-period and they often need to adjust their wealth in several consecutive periods. To construct a more realistic model, it is necessary to investigate multi-period fuzzy portfolio selection problem with interval variables. To our knowledge, there are few researches about multi-period fuzzy portfolio selection problem using interval analysis. The aim of this paper is to utilize the concept of interval numbers in fuzzy set theory to investigate the multi-period fuzzy portfolio selection problem. The contributions of this paper can be summarized as follows. We propose a multi-period fuzzy portfolio selection model with interval analysis, in which the returns, risk and transaction rates of risky assets are characterized by interval numbers. Since the proposed model is an interval programming problem, we use fuzzy decision-making approach to transform it into a crisp form of optimization model. Moreover, we use a hybrid particle swarm optimization algorithm for solution.

The remainder of this paper is organized as follows. For the better understanding of the paper, we will introduce some basic operations and order relations of interval numbers in Section 2. In Section 3, we formulate a multi-period portfolio selection optimization model by using interval analysis. Since the proposed model is equivalent to a fuzzy bi-objective programming problem with interval coefficients on objective and constraints, we first convert it into a crisp form nonlinear programming problem and a particle swarm optimization is designed for solution in Section 4. After that, a numerical example is given to illustrate the application of the proposed model and demonstrate the effectiveness of the designed algorithm in Section 5. Finally, we conclude the paper in Section 6.

2. Basic operations and order relations of interval numbers

In this section, we will briefly review some concepts and results on interval numbers, which we will need in the following sections. Denote the set of all the real number as R. Let $\tilde{a}$ be an interval number. And $\tilde{a}$ can be expressed as the following form

$$\tilde{a} = \left[\underline{a}, \overline{a}\right] = \left\{x : \underline{a} \le x \le \overline{a}, x \in R\right\}. \tag{1}$$

where $\underline{a}$ is the lower bound and $\overline{a}$ is the upper bound of interval $\tilde{a}$, respectively. If $\underline{a} = \overline{a}$, then $\tilde{a}$ is reduced to a real number. The center and width of $\tilde{a}$ are, respectively, defined as

$$m(\tilde{a}) = \frac{\underline{a} + \overline{a}}{2} \text{ and } \omega(\tilde{a}) = \frac{\overline{a} - \underline{a}}{2}.$$

Then, $\tilde{a}$ can also be denoted by its center and width as

$$\begin{aligned}\tilde{a} &= < m(\tilde{a}), \omega(\tilde{a}) > \\ &= \{x : m(\tilde{a}) - \omega(\tilde{a}) \le x \le m(\tilde{a}) + \omega(\tilde{a}), x \in R\}.\end{aligned}$$

Definition 1. Alefeld and Herzberger (1983): Let $\circ \in \{+,-,\times,\div\}$ be a binary operation on R. For any given two interval numbers $\tilde{a}$ and $\tilde{b}$, the binary operation of them is defined as

$$\tilde{a} \circ \tilde{b} = \left\{x \circ y : x \in \tilde{a}, y \in \tilde{b}\right\}, \tag{2}$$

where we assume $0 \notin \tilde{b}$ in the case of division.

Based on the binary operation above, for any given two interval numbers $\tilde{a}$ and $\tilde{b}$, the following relationships hold:

$$\lambda\tilde{a} + \mu\tilde{b} = \left[\lambda\underline{a} + \mu\underline{b}, \lambda\overline{a} + \mu\overline{b}\right], \forall\lambda, \mu \ge 0, \tag{3}$$

$$\lambda\tilde{a} - \mu\tilde{b} = \left[\lambda\underline{a} - \mu\overline{b}, \lambda\overline{a} - \mu\underline{b}\right], \forall\lambda, \mu \ge 0 \tag{4}$$

$$\tilde{a} \cdot \tilde{b} = \left[min\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}, max\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}\right], \tag{5}$$

$$\frac{\tilde{a}}{\tilde{b}} = \left[\frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}}\right], \text{ if } 0 \notin \tilde{b}. \tag{6}$$

An interval number is a special fuzzy number whose membership function takes the value 1 over the interval and 0 anywhere else.

Definition 2. (Ishibuchi and Tanaka (1990)). For any given two interval numbers $\tilde{a}$ and $\tilde{b}$, the order relation between them is defined as

$$\tilde{a} \preccurlyeq \tilde{b} \Leftrightarrow m(\tilde{a}) \le m\left(\tilde{b}\right), \tag{7}$$

if $\overline{a} \le \underline{b}$, the interval inequality relation $\tilde{a} \preccurlyeq \tilde{b}$ is said to be optimistic satisfactory; if $\overline{a} > \underline{b}$, the interval inequality relation $\tilde{a} \preccurlyeq \tilde{b}$ is said to be pessimistic satisfactory.

3. The multi-period portfolio optimization model with interval coefficients

In this section, we will discuss a multi-period portfolio selection problem with interval coefficients, in which the returns, risk and turnover rates of risky assets are qualified by interval numbers. Before modeling, let us first describe the assumptions and notations used in the following sections.

3.1. Assumptions and presentation

We consider a finical market with n risky assets for trading. An investor intends to invest his initial wealth W_0 among the n risky assets