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Quantum-like microeconomics: Statistical model of distribution of investments and production

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ABSTRACT

In this paper we demonstrate that the probabilistic quantum-like (QL) behavior – the Born's rule, interference of probabilities, violation of Bell's inequality, representation of variables by in general noncommutative self-adjoint operators, Schrödinger's dynamics – can be exhibited not only by processes in the micro world, but also in economics. In our approach the QL-behavior is induced not by properties of systems. Here systems (commodities) are macroscopic. They could not be superpositions of two different states. In our approach the QL-behavior of economical statistics is a consequence of the organization of the process of production as well as investments. In particular, Hamiltonian ("financial energy") is determined by rate of return.

1. Introduction

Originally the mathematical formalism of quantum mechanics was developed to serve physical theory of processes in the micro world. However, recently there has been a lot of interest in applications of this mathematical formalism to macroscopic systems and even outside the domain of physics, e.g., in finances, economics, and psychology [1–23], see also [24–38]. From the outset we emphasize that one should sharply distinguish between the mathematical formalism of quantum mechanics and quantum mechanics by itself (as a physical theory). Therefore we are discussing not "applications of quantum mechanics" e.g. in finances or economics, but applications of the quantum mathematical apparatus. To distinguish between the really physical quantum models and other models which might be described by the quantum mathematical apparatus, we shall use the terminology "*quantum-like*" (QL) for the latter case, instead of "quantum".

We consider the calculus of *quantum probabilities* as one of the main distinguishing features of the quantum mathematical apparatus. The crucial point of quantum probability theory is the *Born's rule* for calculation of probabilities. One of the fundamental consequences of this rule is the *interference of probabilities*. Another fundamental consequence is the violation of *Bell's inequality*.

In this paper we demonstrate that the probabilistic QL-behavior – the Born's rule, interference of probabilities, violation of Bell's inequality, representation of variables by in general noncommutative self-adjoint operators – can be exhibited not only by processes in the micro world, but also in economics. In our approach the QL-behavior is induced not by properties of

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systems. Here systems, *commodities*, are macroscopic. They could not be superpositions of two different states. For example, a boat or a car can be either large or small, but never in superposition

$$\psi = c_1 \text{ large} + c_2 \text{ small},$$

where c_1 and c_2 are complex coefficients.[1] The date on a food product can be either expired or not, e.g. meat or fish could not be a superposition of two states:

$$\psi = c_1 \text{ frech} + c_2 \text{ spoiled}.$$

In our approach *the QL-behavior of economical statistics is a consequence of the organization of the process of production as well as investments*.

We consider a toy model of production of boats which have different characteristics interesting for customers, e.g., color, speed, technical equipment (e.g. the presence of radar). Production of boats is distributed in a number of sites in a different parts of the world. These sites differ essentially by labor and investment conditions. In a toy model under consideration, factories which are far from "center" work essentially longer per day. *The working day is proportional to the square of the Euclidean norm*. If a factory is located in the place, given the vector v (pointing from center to this place) then it works T_v hours per day, where T_v is proportional to $\|v\|^2$, the square of the of the Euclidean norm. That's all that we need to organize an industrial process with QL-statistics. The model can be extended by including the factor of investments and consequent amortization needs. Of course, this is still a toy model. However, more realistic realizations might be found. One can interpret the vector v not as determining simply the space location of a production site, but as a vector of generalized coordinates determining some essential *microeconomic characteristics* of the production process at this factory.

Different properties of the product, in our case boats,[2] are represented in general by *noncommutative operators*. In our economical model noncommutativity has no direct relation to incompatibility of variables representing properties. Noncommutativity is exhibition of *complementarity of information about commodities* which is interesting for customers. Here the term complementarity is used in the sense "to complement", to provide additional information. It might be better to use the term "*supplementarity*", as it was proposed in [39], to distance from Bohr's complementarity [25] which has the meaning of mutual exclusivity. In our model supplementary properties of a boat need not be mutually exclusive. For example, they can be color of the boat and its speed. The crucial aspect for supplementarity/noncommutativity is that such properties of commodities could not be reduced to one common property. For example, speed should not be determined by color of the boat and vice versa.

In the last 30 years quantum physics has been involved in great debates initiated by J. Bell on the so-called local realism. There is a rather common opinion that local realism is incompatible with the theoretical formalism of QM [40] as well as experimental results [41,42]. This conclusion of violation of Bell's inequality is well presented outside physics: to philosophers, psychologists, biologists as well as economists. However, experts know well that the real situation is essentially more complicated. The Bell's considerations could not be considered as totally complete from the theoretical viewpoint. A number of hidden assumptions were found which had been used by J. Bell and his followers, see [43–64]. Some of those assumptions are not natural at all from the physical viewpoint. If one assumes that at least one of them is violated (and it is important to notice that "hidden Bell's assumptions" are not verifiable on the present experimental level), then the derivation of Bell's inequality is blocked, see [43–64] for details and debates. Moreover, even the known experimental data contains some anomalies which could not be explained by QM, see [65,66].

The trivial explanation of violation of Bell's inequality in economy is based on nonlocal coupling between processes of production of commodities, e.g. by telephone, fax (but it is purely classical nonlocality).

Another possibility which is essentially more interesting from the viewpoint of economy is to pay attention to an important hidden assumption in Bell's considerations, namely that any system of observables could be realized as random variables on the *same Kolmogorov probability space* [49], see e.g. Accardi [51] or Aerts [67] for details and references. This assumption could be easily violated in economy. Consider a product with a few characteristics. We would like to fix from the very beginning probability distributions of those characteristics in production output. Nobody would be surprised that such prior determination is impossible for arbitrary chosen distributions. If characteristics are constrained, then probability distributions which we would like to fix should be also constrained. In mathematics this problem was solved many years ago by Soviet probabilist Vorobjev [68] (see Hess and Philipp [58] for discussion). This kind of problem has also been studied by Boole (see Pitowsky [69,70] for discussion). In physics this problem was recognized only in QM in the Bell's setup.

[1] Similar problem of states superposition for macroscopic systems arose in attempts to apply QM as physical theory (the quantum reductionism) to modeling of cognitive processes, see [30–33]. In particular, as noted in [31], p. 355, in the quantum reductionist approach a neuron could not process quantum information, because (as a hot and noisy macroscopic system) it cannot be in superposition of two different quantum states, firing and nonfiring, one of which is then selected by a measurement performed in the brain, similarly to a photon passing through or being reflected by a half-silvered mirror, as explained earlier. This impossibility of physical states superposition induced unsolvable problem for Penrose and other supporters of the quantum reductionist approach. They should give up by using macroscopic quantum models for the brain. They tried to proceed from really quantum physical microscale to cognitive processes, i.e. to obtain cognition from behavior of quantum particles composing the brain. It is clear that the latter strategy (the vague quantum reductionism) is totally meaningless in economy. The market strategy of a corporation could not be reduced to behavior of quantum particles composing production sites and commodities produced by those sites.

[2] We consider boats just because it would be easier to operate with this product in considerations on Bell's inequality — to create "nonlocal setup", see Section 11. If one is not interested in Bell's inequality, the car-production may provide even better picture.