Applied Mathematical Modelling 36 (2012) 2429–2445

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Two-stage fuzzy production planning expected value model and its approximation method

Guoqiang Yuan

Fundamental Department, Hebei College of Finance, Baoding 071051, China

ARTICLE INFO

Article history:
Received 10 July 2009
Received in revised form 22 August 2011
Accepted 1 September 2011
Available online 8 September 2011

Keywords:
Production planning
Two-stage fuzzy optimization
Approximation method
Convergence
Simulated annealing

ABSTRACT

This work develops a novel two-stage fuzzy optimization method for solving the multi-product multi-period (MPMP) production planning problem, in which the market demands and some of the inventory costs are assumed to be uncertainty and characterized by fuzzy variables with known possibility distributions. Some basic properties about the MPMP production planning problem are discussed. Since the fuzzy market demands and inventory costs usually have infinite supports, the proposed two-stage fuzzy MPMP production planning problem is an infinite-dimensional optimization problem that cannot be solved directly by conventional numerical solution methods. To overcome this difficulty, this paper adopts an approximation method (AM) to turn the original two-stage fuzzy MPMP production planning problem into a finite-dimensional optimization problem. The convergence about the AM is discussed to ensure the solution quality. After that, we design a heuristic algorithm, which combines the AM and simulated annealing (SA) algorithm, to solve the proposed two-stage fuzzy MPMP production planning problem. Finally, one real case study about a furniture manufacturing company is presented to illustrate the effectiveness and feasibility of the proposed modeling idea and designed algorithm.

1. Introduction

The production planning is to decide what type of, and how much, product should be produced in a production period, it plays an important role in a manufacturing system. The decisions should be made under uncertainty, and uncertainty may be present as randomness and/or fuzziness in the practical production environment. Considering these uncertainties will result in more realistic production planning model. However, the inclusion of uncertainty in the production systems is a more difficult task in terms of modeling and solution. The past four decades shows a growing interest in building models and algorithms for production planning problems such as the material requirements planning (MRP) models [1–3], the aggregate production planning (APP) models [4,5], the production inventory models [6–8], and many others [9–11].

In order to handle probabilistic uncertainty in the production decision systems, some meaningful production planning stochastic models have been proposed in the literature. For example, Hodges and Moore [12] considered a product mix problem with a number of linear resource constraints affecting the decision variables. Beale et al. [13] discussed the product mix problem with a stochastic demand depending on the random demands. Since the work of Dantzig [14], two-stage programs under stochastic environment have well extended and greatly developed in different disciplines, such as operations research, management science, control theory and artificial intelligence. In two-stage stochastic production problem, Gopalan and Anantharaman [15] examined the transient and steady-state characteristics of a two-stage transfer-line production system subject to inter-stage and end inspections and end buffer; Beraldi et al. [16] proposed a two-stage stochastic integer

E-mail addresses: yggqq@126.com, yggqq@163.com

0307-904X/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2011.09.001

programming model for the integrated optimization of power production and trading which include a specific measure accounting for risk management; Sayarshad and Moghaddam [17] proposed a two-stage stochastic optimization formulation and solution procedure for optimizing the fleet size and freight car allocation under uncertainty demands. The readers who are interested in detailed discussion about two-stage stochastic programming may refer to the books [18,19].

Since the pioneering work of Zadeh [20], possibility theory was being perfected and became a strong tool to deal with possibilistic uncertainty in fuzzy decision systems [21–23]. Many researchers applied the theory successfully to fuzzy optimizations and effectively dealt with many practical problems in the past three decades [24–27]. More importantly, possibility theory also plays a key role for describing and processing fuzzy information in the realistic production decision systems. Therefore, fuzzy production planning problems attract many researchers′ interests [28,29]. Among them, Shih [28] applied three fuzzy linear programming models to solving transportation planning problem; Sharma et al. [29] presented a fuzzy goal programming model for handling fuzzy goal such as production and income of farmers in rural development planning. Although possibility theory is very popular and widely used in fuzzy community, the recent studies [30–33] show that it is credibility measure instead of possibility measure that plays the role of probability measure in fuzzy decision systems, and an axiomatic approach based on credibility measure, called credibility theory, was developed by the motivation of the fact [34]. In addition, credibility theory has attracted much attention and been applied in many fields to deal with incomplete and uncertain situation. For example, Zhang et al. [35] considered a fuzzy age-dependent replacement policy in which the lifetimes of components were treated as fuzzy variables, and the long-term expected cost per unit time was minimized; Liu [36] considered a new class of two-stage fuzzy programming problem in 2005. Subsequently, he studied the basic properties of the proposed programming problems and discussed convergence of approximating a recourse function. Finally, he designed a heuristic algorithm to solve the proposed two-stage fuzzy programming problem; Lan et al. [37] presented a class of single-stage fuzzy production planning problem with credibility objective. However in contrast to previous stochastic and fuzzy production planning problem, credibility theory and two-stage fuzzy optimization method [36] to production planning problems have not been studied extensively in the literature.

The purpose of this paper is to apply credibility theory and two-stage fuzzy programming theory to practical production planning problem and presents a new class of two-stage fuzzy MPMP production planning expected value model. Besides, some basic properties about the proposed production planning problem are discussed in this paper. The two-stage optimization methods were also used in the literature by a number of researchers. For example, Bakir and Byrne [38] considered an multi-product multi-period production planning model with stochastic demand. Subsequently, they developed a new demand stochastic linear programming model based mainly on the theoretical presentation of two-stage deterministic equivalents. In this paper, motivated by two-stage stochastic optimization [18], we define two-stage optimization using credibility measure instead of probability measure for our MPMP production planning problem. The proposed production planning problem in this paper is more general than [38], in which the market demands and some of the inventory costs are assumed to be uncertainty and characterized by fuzzy variables with known possibility distributions. Since the fuzzy market demands and fuzzy inventory costs usually have infinite supports, the proposed two-stage fuzzy production planning problem belongs to an infinite-dimensional optimization problem that cannot be solved directly. To overcome this difficulty, this paper considers an approximation method (AM) [39] for the original two-stage fuzzy MPMP production planning problem, and turns it to a finite-dimensional optimization problem. The convergence of the AM is also discussed in this paper. Since the approximating MPMP production planning problem is neither linear nor convex, conventional optimization algorithms cannot be applied in this paper. Therefore, we will design a heuristic algorithm, which combines the AM and SA algorithm, to solve the proposed two-stage fuzzy MPMP production planning problem. To demonstrate the practical relevance of our research, we design a furniture manufacturing problem as a five-product five-period production planning problem in this paper.

The rest of this paper is organized as follows. Firstly, we will recall some basic concepts in Section 2, and then propose a new type of two-stage fuzzy MPMP production planning expected value model in Section 3. Subsequently, some basic properties about the two-stage fuzzy MPMP production planning problem are discussed in Section 4. In Section 5, we employ the AM to the expected value function of two-stage fuzzy MPMP production planning problem, and deal with the convergence of AM. The convergent result facilitates us to incorporate AM and SA algorithm to solve the proposed MPMP production planning problem in Section 5. To apply the proposed approach to a practical MPMP production planning problem, a real case study about a furniture manufacturing company is given in Section 6 to illustrate the feasibility and effectiveness of the designed heuristic algorithm. Section 7 summarizes the main results in this paper and points out our future research.

2. Preliminaries

The concept of fuzzy set was initiated by Zadeh [40] via membership function in 1965. In order to measure a fuzzy event, Zadeh [20] proposed the concept of possibility measure in 1978. Although possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu and Liu [30] presented the concept of credibility measure in 2002. Based on credibility measure, an axiomatic approach, called credibility theory [34,41] was studied extensively. Subsequently, we first recall some basic concepts in the following section.

Given a universe Γ, $\mathcal{P}(\Gamma)$ is the power set of Γ, and Pos is a set function define on $\mathcal{P}(\Gamma)$. The set function Pos is called a possibility measure [23,42] if it satisfies the following conditions: